ECON 6090
Section 6
Gabe Sekeres
Fall 2025

Material

Welfare

The core question here: How can we tell when consumers will be better or worse off for a given price change?

Definition. A *money metric indirect utility function* is an indirect utility function of the form $e(p', V(p, w))$ for some fixed p'. Since $e(\cdot)$ is strictly increasing in $V(\cdot)$, it is a valid indirect utility function, and it is additionally invariant under positive affine transformation of u.

Assumption 1. *Let's consider a change in prices from p^0 to p^1, fixing wealth at w. Let $\bar{u}^0 = V(p^0, w)$ and $\bar{u}^1 = V(p^1, w)$.*

Definition. The *compensating variation* is defined as

$$CV(p^0, p^1, w) = e(p^1, \bar{u}^1) - e(p^1, \bar{u}^0) = w - e(p^1, \bar{u}^0)$$

The *equivalent variation* is defined as

$$EV(p^0, p^1, w) = e(p^0, \bar{u}^1) - e(p^0, \bar{u}^0) = e(p^0, \bar{u}^1) - w$$

Question. These are two of the four possible differences. What is the intuition behind why we use these two rather than the other two? What do the other two represent?

Proposition 1. *Suppose the price of only one good changes, and let that good have index 1. Then*

$$CV(p^0, p^1, w) = \int_{p_1^1}^{p_1^0} h_1(p_1, p_{-1}, \bar{u}^0)\, \partial p_1 \qquad ; \qquad EV(p^0, p^1, w) = \int_{p_1^1}^{p_1^0} h_1(p_1, p_{-1}, \bar{u}^1)\, \partial p_1$$

Exercise. What is the intuition behind the proof here? It can be pretty directly shown from Shephard's Lemma, but why does this hold?

Aggregation

The core question here: When can, instead of considering many consumers each with wealth w_i, can we consider a single consumer with wealth $W = \sum_i w_i$?

Theorem 1. *We have a single positive representative consumer if and only if consumers' indirect utility functions take the* Gorman form

$$V_i(p, w_i) = a_i(p) + b(p) \cdot w_i$$

where $b(p)$ is the same for all consumers.
Remark. Basically everything goes through here as you'd expect. The aggregate Marshallian demand will be

$$x(p, W) = \sum_i x_i(p, w_i)$$

and all the other functions will be aggregated the same way.

Producer Theory Preliminaries

Definition. Assume that we have L commodities. A *production plan* is a vector $y \in \mathbb{R}^L$ with *net inputs* $y_i < 0$ and *net outputs* $y_j > 0$. We define the *production set* as the set of *feasible* production plans, and call it $Y \subseteq \mathbb{R}^L$. Any $y \in Y$ is feasible, any $y \notin Y$ is not. We will often describe production sets with the *production function* (or, more formally, *transformation function*) $F(\cdot)$, which is defined as having the property that

$$Y = \{y \in \mathbb{R}^L : F(y) \leq 0\}$$

If we have a single-output technology, we call the production function $f(z)$, for inputs $z \in \mathbb{R}^{L-1}$, and set output $q \leq f(z)$.
Remark. MWG describes the production set as the *primitive datum* of the theory. Most of the time, in practice, we will think of production functions and other smoothing tools that make our very pretty theorems work. It's always important to remember, though, that the production functions are defined as the border of the production set, and not *vice versa*.

Informally, we will tend to work in the single-output case. In that case, we will define the *production function* as $f : \mathbb{R}_+^{L-1} \to \mathbb{R}_+$ as

$$f(z) = \max_z q \text{ s.t. } (q, -z) \in Y$$

We will define the *input requirement set* as

$$V(q) = \{z \in \mathbb{R}_+^{L-1} : (q, -z) \in Y\}$$

and we will define the *isoquant* as

$$Q(q) = \{z \in \mathbb{R}_+^{L-1} : z \in V(q) \text{ and } z \notin V(q') \text{ for any } q' > q\}$$

We will think about the intuition of these definitions more than the formalities.

Practice Questions

1. In this question, we seek to investigate the relation between equivalent variations and compensating variations. Consider an economy with two goods: good 1 and good 2 respectively. The initial price vector is $p^0 = (p_1^0, p_2^0)$.

   (a) (adapted from 2002 Prelim 2) Suppose we only increase the price of good 1 and get a new price vector $p^1 = (p_1^1, p_2^0)$, where $p_1^1 > p_1^0$. Discuss in case where x_1 is normal good or inferior good and compare $CV\left(p^0, p^1, w\right)$ and $EV\left(p^0, p^1, w\right)$

   (b) Suppose a consumer's preference is quasi-linear in y with $u(x, y) = x^\alpha + y$, where $\alpha \in (0,1), x \in \mathbb{R}_+, y \in (-\infty, +\infty)$. Now instead of only changing p_1, we allow both p_1 and p_2 changes, the new price vector is $p^1 = (p_1^1, p_2^1)$. Compare $CV\left(p^0, p^1, w\right)$ and $EV\left(p^0, p^1, w\right)$ in this case. How will your answer change from (a)?

2. (Adapted from MWG) Recall that the Gorman form is $v^i\left(p, w^i\right) = a^i(p) + b(p)w^i$. Verify that Gorman form implies a linear wealth expansion path, which means the Walrasian demand function is a linear function in wealth.

3. For each of the following utility functions, does a positive representative consumer exist?

   (a) $u^i(x, y) = x^{a^i} y^{1-a^i}$

   (b) $u^i\left(x_1, \ldots, x_N\right) = \sum_{n=1}^N \alpha_n \log x_n, \alpha_n > 0$

   (c) (2022 Prelim 2) $u^i(x, y) = ln(x) + y$, where wealth $w^i > 1$ and price $p_x > 0$ and $p_y = 1$.