**ECON 6090**

*Section 4*

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# Material

We're only going to cover a single thing today, but it encompasses everything from consumer theory:

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
x(p,w) \arrow[rr, bend right, "V = u(X)"'] \arrow[dd, leftrightarrow, bend left, "\begin{array}{l} x = h(p,V) \\ h = x(p,e) \end{array}"] \arrow[dd, leftrightarrow, dotted, bend right, "\text{Slutsky: } \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w} x_j = \frac{\partial h_i}{\partial p_j}"'] & & V(p,w) \arrow[ll, bend right, "\text{Roy: } x = \frac{-\nabla_p V}{\partial V/\partial w}"'] \arrow[dd, leftrightarrow, "\begin{array}{l} V(p,e) = \bar{u} \\ e(p,V) = w \end{array}"] \\
& & \\
h(p,\bar{u}) \arrow[rr, bend right, "\text{Adding-up: } e = p \cdot h"'] & & e(p,\bar{u}) \arrow[ll, bend right, "\text{Shephard: } h = \nabla_p e"']
\end{tikzcd}
$$

Figure 1: Relationships Between UMP and EMP

# Practice Question

1. **June 2025 Microeconomics Q Exam (Part II)** Consider a consumer who has utility $u(x_1, \dots, x_L, h)$ over the amounts of consumption $(x_1, \dots, x_L)$ of goods $1, \dots, L$, and hours worked $h$. The utility function $u(\cdot)$ is continuous and strictly increasing in each $x_\ell$, and is weakly decreasing in $h$. Prices are given by $p = (p_1, \dots, p_L)$. The wage rate is normalized to 1 throughout, so that the consumer can spend amount $h$ to purchase consumption goods. Consumption and labor hours have to be non-negative, but assume that there is no upper bound on labor hours $h$.

   (a) Write down the "work-minimization problem": the problem of minimizing the hours of work subject to achieving a given level of utility and subject to the budget constraint.

   Call the value $h^\star$ of this problem the "minimal-work function". Does it have similar properties to the expenditure function? In particular:

   (b) Is the "minimal-work function" non-decreasing in $p_i$ for $i \in \{1, \dots, L\}$? (Hint: If $(x_1, \dots, x_L, h)$ satisfies all constraints under higher prices, would it still satisfy all constraints under lower prices?).

   (c) Is the "minimal-work function" homogeneous of degree 1 in $p$? (Hint: if you doubled the prices of all goods, would the minimal-work-hours double? Try e.g. with $u(x_1, h) = x_1^2/h$.)

   (d) What would your answer to (b) and (c) be if you knew that the utility function is constant in work hours $h$?

   Prove your answers. For negative answers you can provide a counter-example. You do not need to re-prove known properties of the standard expenditure function.