ECON 6090-Microeconomic Theory. TA Section 10

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December 6, 2024

In Section notes

Subjective Expected Utility: Anscombe-Aumann

1. S: States.
2. X: Outcomes.
3. $\mathcal{P}$: All possible distributions over X.
4. H: acts[1] $= \{h | h : S \to \mathcal{P}\}$.
5. π: prior/belief over states.

SEU: one belief $\pi \in \Delta(S) = \mathcal{P}$

$$SEU(h) = \sum_{s \in S} \pi(s) \sum_{x \in X} h(x|s) u(x)$$

Objective: $\max_h SEU(h)$.

Beyond SEU (Gilboa-Schmeidler type)

We only have a set of beliefs Q. We want to max-min the utility.

$$\max_h \{\min_{\pi \in Q} [\sum_{s \in S} \pi(s) \sum_{x \in X} h(x|s) u(x)]\}$$

Exercises

(Qualifying 2022)

We know

- h: bet on head
- t: bet on tail
- n: do not bet

We define our states as $S = \{H, T\}$, and our acts as, $Acts = \{h, t, n\}$. Then,

$$P(head|H) = \frac{2}{3}$$

$$P(tail|T) = \frac{2}{3}$$

(a) If $\pi(T) = \pi(H) = \frac{1}{2}$, we have the following scenarios,

[1] Reminder that in in the Savage famework, $h : S \to X$

1) Accept bet on head:

$$SEU(h) = \pi(H)[P(head|H)u(\beta w) + P(tail|H)u(\alpha w)] + \pi(T)[P(head|T)u(\beta w) + P(tail|T)u(\alpha w)]$$

$$SEU(h) = \frac{1}{2}[\frac{2}{3}ln(\beta w) + \frac{1}{3}ln(\alpha w)] + \frac{1}{2}[\frac{1}{3}ln(\beta w) + \frac{2}{3}ln(\alpha w)]$$
$$= \frac{1}{2}ln(\beta\alpha w^2)$$

2) Accept bet on tail:

$$SEU(t) = \frac{1}{2}[\frac{1}{3}ln(\beta w) + \frac{2}{3}ln(\alpha w)] + \frac{1}{2}[\frac{2}{3}ln(\beta w) + \frac{1}{3}ln(\alpha w)]$$

$$= \frac{1}{2}ln(\beta\alpha w^2)$$

3) Do not bet:

$$SEU(n) = u(w) = ln(w)$$

Therefore, the person would be indifferent between accepting and rejecting the bet when,

$$SEU(n) = \max\{SEU(t), SEU(h)\}$$

Which means,

$$ln(w) = \frac{1}{2}ln(\beta\alpha w^2)$$
$$\implies \alpha\beta = 1$$

(b) Now the individual is an ambiguity averse decision maker (Gilboa-Schmeidler type). This means that the individual wants to minimize uncertainty by analizing the worst case scenario. The problem becomes,

$$SEU(h) = \min_{\pi(H)\in[0,1]} \pi(H)[\frac{2}{3}ln(\beta w) + \frac{1}{3}ln(\alpha w)] + (1-\pi(H))[\frac{1}{3}ln(\beta w) + \frac{2}{3}ln(\alpha w)]$$

$$= \frac{1}{3}ln(\beta w) + \frac{2}{3}ln(\alpha w)$$

That is the case when $\pi(H) = 0$. Similarly,

$$SEU(t) = \frac{1}{3}ln(\beta w) + \frac{2}{3}ln(\alpha w)$$

And,

$$SEU(n) = ln(w)$$

Therefore, the person would be indifferent between accepting and rejecting the bet when,

$$SEU(n) = \max\{SEU(t), SEU(h)\}$$

Which means,

$$ln(w) = \frac{1}{3}ln(\beta w) + \frac{2}{3}ln(\alpha w)$$
$$\implies (\alpha^2\beta)^{\frac{1}{3}} = 1$$
$$\implies \alpha^2\beta = 1$$

(c) If we change $\pi(H)$ from $\frac{1}{2}$ to $\frac{1}{9}$ in part (a), we get,

$$SEU(h) = \frac{1}{2}[\frac{9}{10}ln(\beta w) + \frac{1}{10}ln(\alpha w)] + \frac{1}{2}[\frac{1}{10}ln(\beta w) + \frac{9}{10}ln(\alpha w)]$$

$$= \frac{1}{2}ln(\beta\alpha w^2)$$

And indifference between betting and not betting implies,

$$\alpha\beta = 1$$

Which is the same result as we got in part (a).
Now, if we change $\pi(H)$ from $\frac{1}{2}$ to $\frac{1}{9}$ in part (b), we get,

$$SEU(h) = \frac{1}{10}ln(\beta w) + \frac{9}{10}ln(\alpha w)$$

And indifference implies,

$$\alpha^9\beta = 1$$

Which is larger than the result we got in part (b). This happens because now the worst case scenario "becomes worst" in the sense that it is more likely to happen.

(Final 2021)

(a) If $\pi(A) = \pi(B) = \frac{1}{2}$, we have the following scenarios,

1) Accept bet on head:

$$SEU(H) = \pi(A)[P(head|A)u(w+1) + P(tail|A)u(w-1)] + \pi(B)[P(head|B)u(w+1) + P(tail|B)u(w-1)]$$

$$SEU(H) = \frac{1}{2}[\frac{1}{3}(w+1) + \frac{2}{3}(w-1)] + \frac{1}{2}[\frac{3}{4}(w+1) + \frac{1}{4}(w-1)]$$

$$SEU(H) = w + \frac{1}{12}$$

2) Accept bet on tail:

$$SEU(T) = \pi(A)[P(head|A)u(w-1) + P(tail|A)u(w+1)] + \pi(B)[P(head|B)u(w-1) + P(tail|B)u(w+1)]$$

$$SEU(T) = \frac{1}{2}[\frac{1}{3}(w-1) + \frac{2}{3}(w+1)] + \frac{1}{2}[\frac{3}{4}(w-1) + \frac{1}{4}(w+1)]$$

$$SEU(T) = w - \frac{1}{12}$$

3) Do not bet:

$$SEU(n) = u(w) = w$$

Since,

$$SEU(n) = w < w + \frac{1}{12} = \max\{SEU(H), SEU(T)\}$$

The individual will bet. Specifically, on H.

(b) Now that the individual is a Gilboa-Schmeidler ambiguity averse person[2], the problem becomes,

$$SEU(H) = \min_{\pi(A)\in[1/3,2/3]} \pi(A)[\frac{1}{3}(w+1) + \frac{2}{3}(w-1)] + (1-\pi(A))[\frac{3}{4}(w+1) + \frac{1}{4}(w-1)]$$

The expression is minimized when $\pi(A) = 2/3$,

$$SEU(H) = \frac{2}{3}[\frac{1}{3}(w+1) + \frac{2}{3}(w-1)] + \frac{1}{3}[\frac{3}{4}(w+1) + \frac{1}{4}(w-1)]$$

$$= w - \frac{1}{18}$$

And,

$$SEU(T) = \min_{\pi(A)\in[1/3,2/3]} \pi(A)[\frac{1}{3}(w-1) + \frac{2}{3}(w+1)] + (1-\pi(A))[\frac{3}{4}(w-1) + \frac{1}{4}(w+1)]$$

$$= w - \frac{2}{9}$$

Since,

$$SEU(n) = w > w - \frac{1}{18} = \max\{SEU(H), SEU(T)\}$$

The individual will not bet.

[2]The person analyzes worst case scenario.