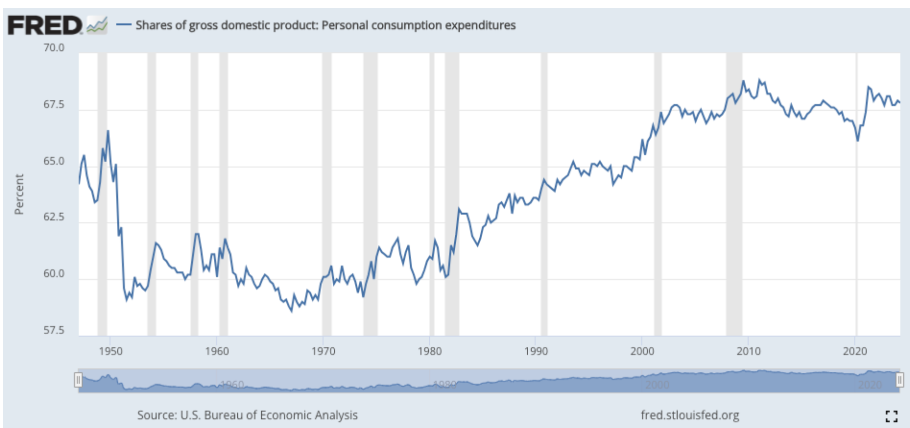
Problem Set 3

September 2024

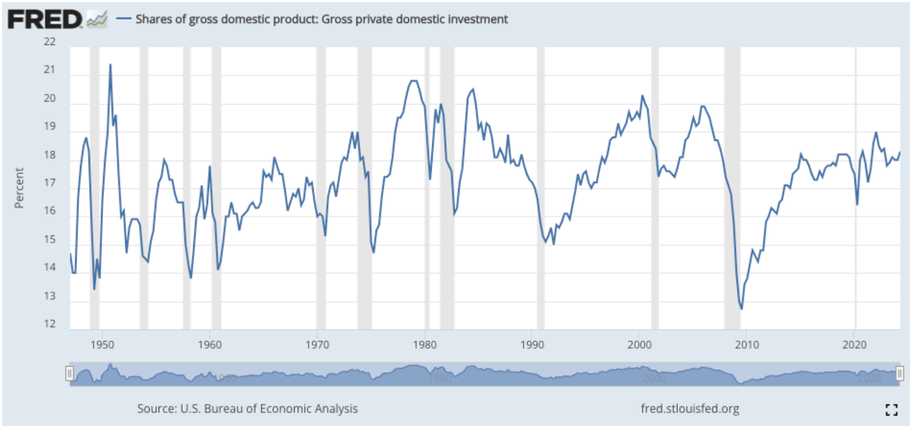
1 Problem 1

1.1 Consumption as a fraction of GDP



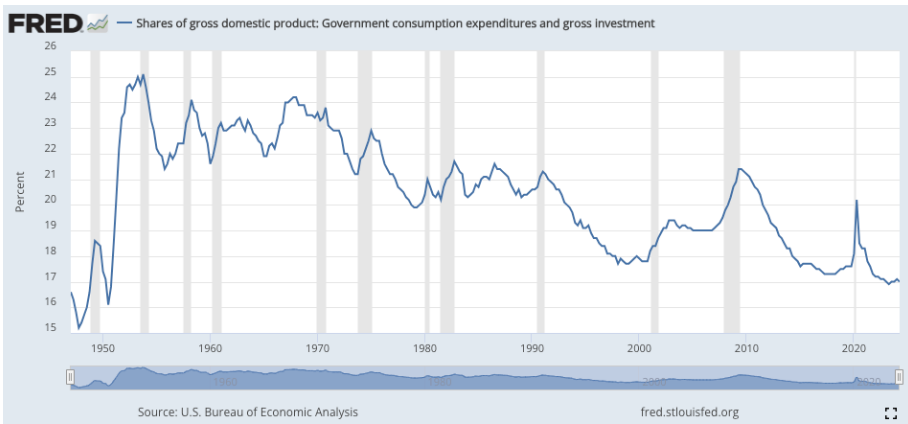
Over the past 50 years, consumption has followed several key trends, typically ranging between 60-67.5% of GDP since 1970. Various economic events have caused significant shifts in consumption patterns. In the 1970s, oil crises and rising inflation led to slow economic growth, stagnating wages, and reduced purchasing power, which limited consumption growth. In the 1980s, economic reforms like tax cuts, deregulation, and the growing use of credit cards helped boost private consumption. The 1990s saw a strong rise in consumption, driven by technological innovation and globalization, with U.S. consumption as a share of GDP reaching around 67%, levels not seen since the post-World War II boom of the 1950s. In the 2000s, consumer spending continued to increase gradually until the 2008 financial crisis, during which the housing market collapse and low interest rates significantly impacted spending. However, even at the lowest point of the crisis, consumption as a share of GDP remained relatively high, near the levels seen in 2005. During the 2010s, consumption growth slowed, likely due to wage stagnation and rising inequality, which kept the consumption-to-GDP ratio stable. This pattern persisted until the COVID-19 pandemic when consumption plummeted due to consumer uncertainty. Since then, consumer demand has rebounded strongly, though inflation, supply chain disruptions, and rising interest rates have introduced new challenges to consumption patterns.

1.2 Investment as a fraction of GDP



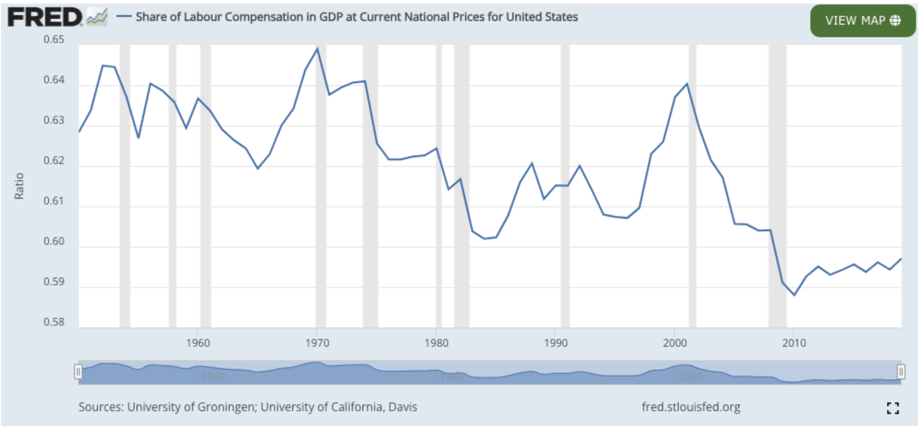
Investment as a share of GDP tends to be more volatile than consumption and is closely tied to business cycles. It typically rises during periods of economic expansion and falls during recessions. In the U.S., investment as a share of GDP usually averages between 14% and 20%. There were sharp declines during the recessions of the 1970s and 1980s for reasons mentioned earlier. The tech boom of the 1990s drove a surge in business investment, peaking in the early 2000s before sharply declining due to the bursting of the dot-com bubble. At its peak, investment reached about 20% of GDP before dropping below 18%. Investment recovered quickly until the 2008 financial crisis caused a significant drop, falling below 13%. The recovery following 2008 was gradual. Interestingly, the decline in investment during the COVID-19 pandemic was not as steep as one might expect, given the typical volatility of investment. It rebounded relatively quickly, likely due to government stimulus and pent-up demand.

1.3 Government (federal and all levels) spending as a fraction of GDP



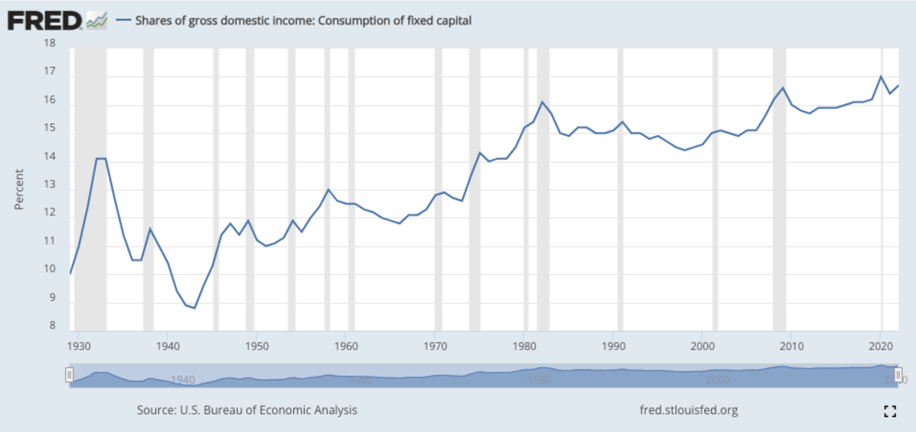
Since the 1950s, government expenditures have grown steadily, likely due to post-WWII rebuilding efforts and infrastructure expansion, however has dropped as a share of GDP. As government spending appears to be highly tied to wars, this figure highlights slight but notable increases during the Vietnam and Korean Wars in the 1960s-70s. One of the few prolonged reductions in government spending occurred in the 1990s, as the U.S. government focused on reducing the deficit and curbing expenditures. However, in the early 2000s, spending began to rise again, driven in part by the wars in Afghanistan and Iraq. Another significant increase occurred following the 2008 financial crisis, as stimulus packages and bailouts were introduced to stabilize the economy. The surge in spending during the COVID-19 pandemic was unprecedented, with multiple stimulus packages pushing government expenditures to levels not seen since the mid-20th century (although as a share of GDP, this level of spending appears less significant.

1.4 Payments to labor as a share of GDP



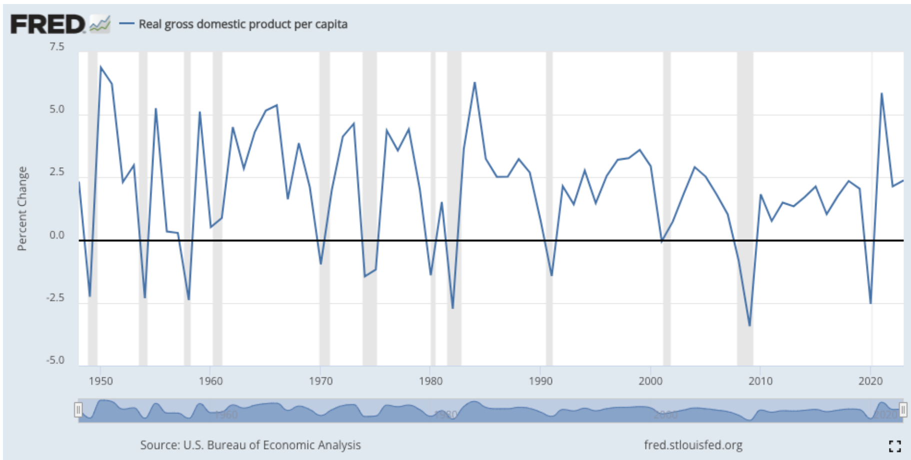
Labor share represents the portion of a nation's total economic output that is allocated to workers. Post WWII, labor's share of the GDP reached high levels due to growth in manufacturing and strong union power. From the 1950s-'70s, labor share remained stable. However, in the 1970's, and continuing into the 1980's, companies began to outsource their labor, weakening domestic labor demand, while technological advancements made certain high-paying manufacturing jobs obsolete. The 1990s-2000s can largely be characterized by declines in labor share due to corporate profits increasing while wages remained sluggish. Since the 2008 financial crisis, the labor share of GDP dropped even further. The recovery period saw an increase in corporate profits and stock market performance while wages remained flat. The share of income paid to labor never recovered.

1.5 Payments to capital as a share of GDP



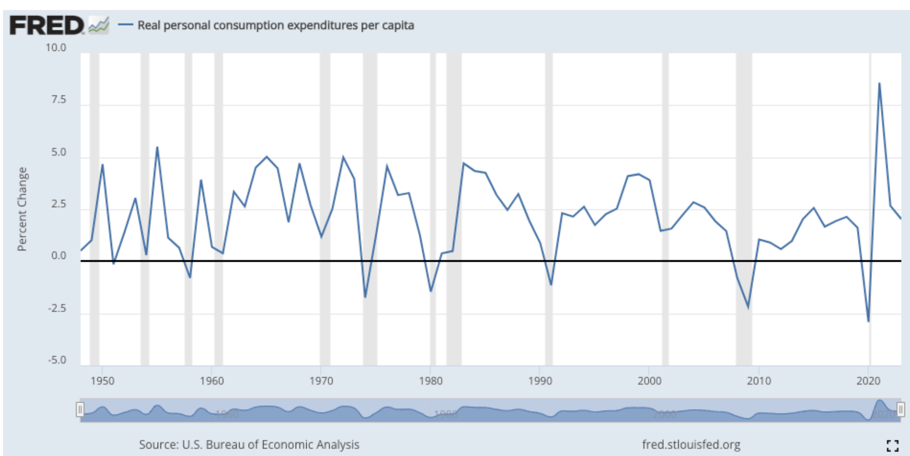
Payments to capital as share of GDP represents the portion of national income that goes to the owners of capital in the form of profits, interest, and rent. As the share of labor and the share of capital together account for much of the total output of the economy, the capital share is the complement of the labor share.

1.6 The growth rate of output per capita



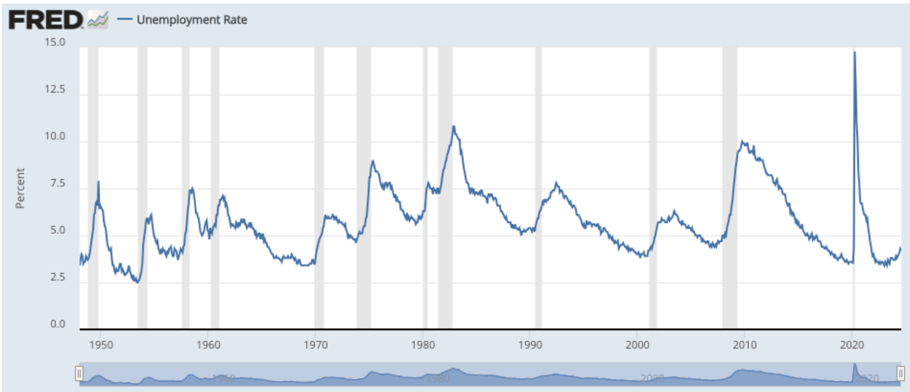
Real GDP per capita has overall grown at a rate of about 2 percent with fluctuations due to business cycles – most notably with larger fluctuations and negative growth during the early 1980's, the 2008 recession, and the pandemic. The average growth rate in GDP has been around 1.5-2% per year since the 2000s, a decline from 2.5-3% enjoyed in the 1980's 90's.

1.7 The growth rate of consumption per capita



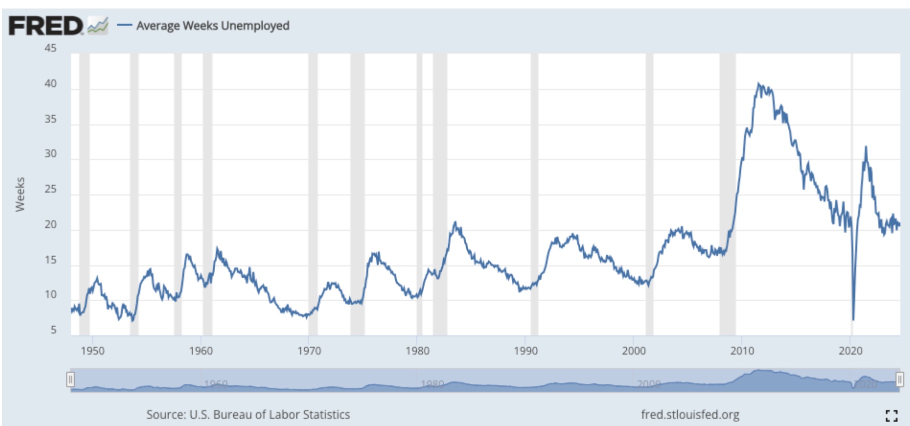
The measure of personal consumption expenditures per capita, or the amount of consumption per person, tends to follow the same trend as real GDP per capita, with the same fluctuations in business cycles and the same average growth rate of around 2% per year, however, appears to be less volatile than GDP per capita, potentially due to consumer's preferences for consumption smoothing.

1.8 Civilian unemployment rate



The average civilian unemployment rate in the US appears to fluctuate between 4 and 7%, on average. Unemployment appears to be highly inversely correlated with macroeconomic fluctuations, with downturns increasing unemployment and booms decreasing unemployment. The pandemic caused an unprecedented peak in unemployment at a rate of nearly 15%, with the second highest peak being from the 1980's recession at a rate of around 11% unemployment.

1.9 Average duration unemployment



The average number of weeks unemployed was fairly steady (depending on the current point of the economic cycle) at around 10 weeks during economic booms, and around 15-20 weeks during recessions. However, the massive unemployment during the 2008 recession took longer to recover from, and so unemployment lasted for nearly 40 weeks during that time, coming steadily downward until reaching a sharp trough in 2020 – the massive unemployment seen during the pandemic was short-lived, with the average time for being unemployed only lasting at around 5 weeks.

2 Problem 2

The planner's optimization problem as a dynamic programming problem is as follows:

$$V_n(k) = max\{ln(f(k) - k') + \beta V_{n-1}(k')\} \tag{1}$$

Where our control variables are k, and our state variables are α, β, and δ.

We know that the given $\alpha = 0.3$ and that the given $\beta = 0.6$. The given capital depreciation rate is $\delta = 0.75$

$$f(k) = k^\alpha + (1 - \delta)k \tag{2}$$

$$f(k') = \alpha k^{\alpha - 1} + 1 - \delta \tag{3}$$

Using value function iteration and taking our first order conditions with respect to k yields:

$$\frac{\delta V(k)}{\delta k'} = \frac{-1}{f(k) - k'} + \beta V'(k) \tag{4}$$

$$\frac{1}{f(k) - k'} = \beta V'(k') \tag{5}$$

By the envelope theorem,

$$V'(k) = \frac{f'(k)}{f(k) - k'} \tag{6}$$

$$V'(k') = \frac{f'(k')}{f(k') - k''} \tag{7}$$

By equations (5) and (7), we can obtain:

$$\beta \frac{f'(k')}{f(k') - k''} = \frac{1}{f(k) - k'} \tag{8}$$

$$\beta f'(k')(f(k) - k') = f(k') - k'' \tag{9}$$

By replacing f(k) with its functional form, we find:

$$\beta(k^\alpha + (1 - \delta)k - k')(\alpha k^{\alpha - 1} + 1 - \delta) = k'^\alpha + (1 - \delta)k' - k'' \tag{10}$$

And by replacing δ with 0.75, we have:

$$\beta(k^\alpha + (1/4)k - k')(\alpha k^{\alpha - 1} + 1/4) = k'^\alpha + (1/4)k' - k'' \tag{11}$$

At the steady state $k = k' = k''$, the above simply boils down to:

$$\beta(\alpha k^{*\alpha - 1} + (1/4)) = 1 \tag{12}$$

$$\alpha k^{*\alpha-1} = \frac{1}{\beta} - \frac{1}{4} \tag{13}$$

$$k^* = (\frac{1}{\alpha}(\frac{1}{\beta} - \frac{1}{4}))^{\frac{1}{\alpha-1}} \tag{14}$$

With an $\alpha = 0.3$ and $\beta = 0.6$, our steady state of capital is .11089. Figure 1, below, shows the same value at the intersection of the Policy Function and the 45 degree line when calculated via MatLab.

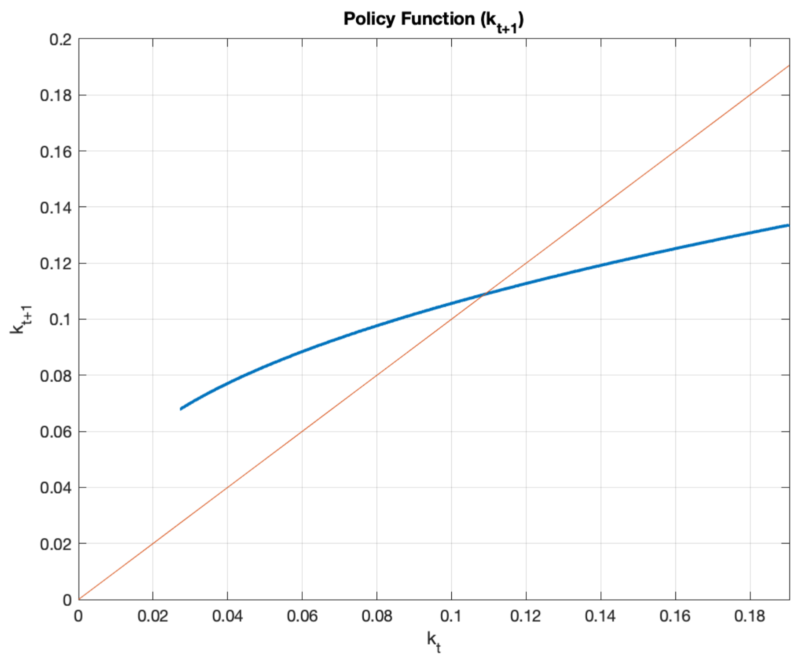
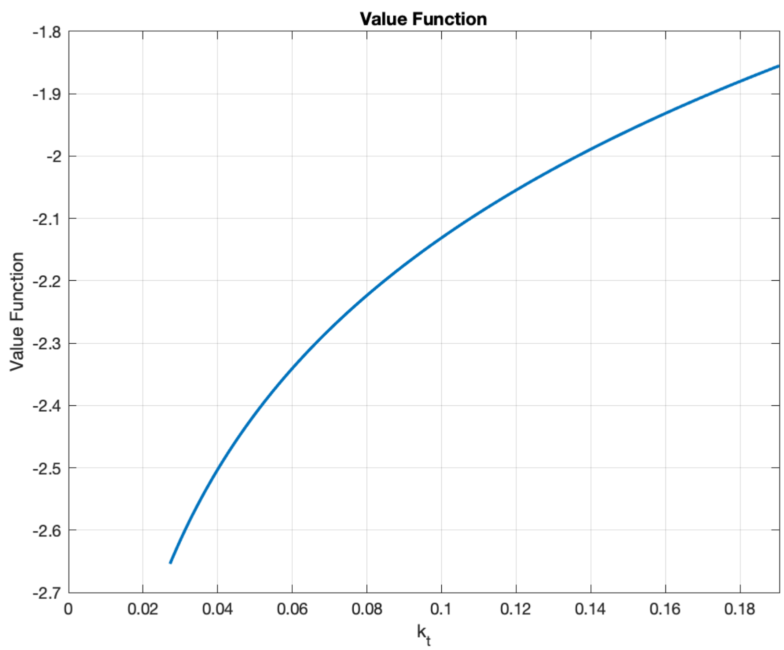


Figure 2, on the following page, provides the solution of the Value Function in this economy.


Value Function
Value Function
k_t
-1.8
-1.9
-2
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18

MatLab code is as follows:

```
%%
% Change current folder


%
clc; %clears the command window
clear all; %clears variables
close all; %close all figures
format compact %Set the output format to the short engineering format with compact line spacing
% Time recording
tic %tic works with the toc function to measure elapsed time, the tic function records the current time


% Parameters setting
alpha = 0.3; % share of capital from total output
beta = 0.6; % discount factor
its = 1; % Initialize the number of iterations for value function iteration
diff = 1; % Initial difference between the old and new value function
tol = 1e-6; % Tolerance level to stop the iteration (controls the accuracy of the solution)
u = @(c) (c>0).*log(c) + (c<=0).*(-1e18); % utility function with Inada! %(c>0) indicates which values are greater than zero. % For example: A = [-3 -1 0 9 4 3 2]; The output of the command b = (A>0) is: b = [0 0 0 1 1 1 1]
kss = ((1/alpha)*((1/beta)-(1/4)))^(1/(alpha-1)) ; % steady state capital stock
nk = 1000; % number of data points in the the capital grid
kmin = 0.25*kss; % minimum value in the capital grid ... 75% lower than the steady state
kmax = 1.75*kss; % maximum value in the capital grid ... 75% more than the steady state
kgrid = linspace(kmin,kmax,nk); % capital grid
%y = linspace(x1,x2,n) generates n points. The spacing between the points is (x2- x1)/(n-1).
val_fun = zeros(1,nk); % initial value functions
pol_fun_idx = zeros(1,nk); % indexes for the policy function
```

```
%%
%Value Function Iteration
while diff>tol
for i=1:length(kgrid)
c = ((kgrid(i)^alpha)+0.25*kgrid(i))-kgrid;
[val_new(i), pol_fun_idx(i)] = max(u(c)+beta*val_fun);
     % Bellman equation
end

diff= max(abs((val_new-val_fun)));
val_fun=val_new;
its = its+1;
end

pol_fun = kgrid(pol_fun_idx); % This collects the
   points on the grid that resulted in the maximal
   value function
cons = (kgrid.^alpha)-pol_fun;

%% Plots
% Plotting the value function and the policy function
figure(1)
plot(kgrid,pol_fun,'linewidth',1.8); title('Policy
   Function (k_{t+1})'); ...
xlabel('k_t'); ylabel('k_{t+1}'); grid on ; hold on;
   plot([0 kmax],[0
kmax]); ...
xlim([0 kmax]); saveas(gcf,'pol_fun_k.png')
%plot(X,Y) plots a 2-D line plot of the data in Y
   versus the corresponding values in X.

%% Plots
% Plotting the value function
figure(2)
plot(kgrid,val_fun,'linewidth',1.8); title('Value
   Function'); ...
xlabel('k_t'); ylabel('Value Function'); grid on; ...
xlim([0 kmax]); saveas(gcf, 'val_fun_k.png')

toc % The toc function uses the recorded value to
   calculate the elapsed time.
```