Continuous-Time Growth Theory

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Wharton

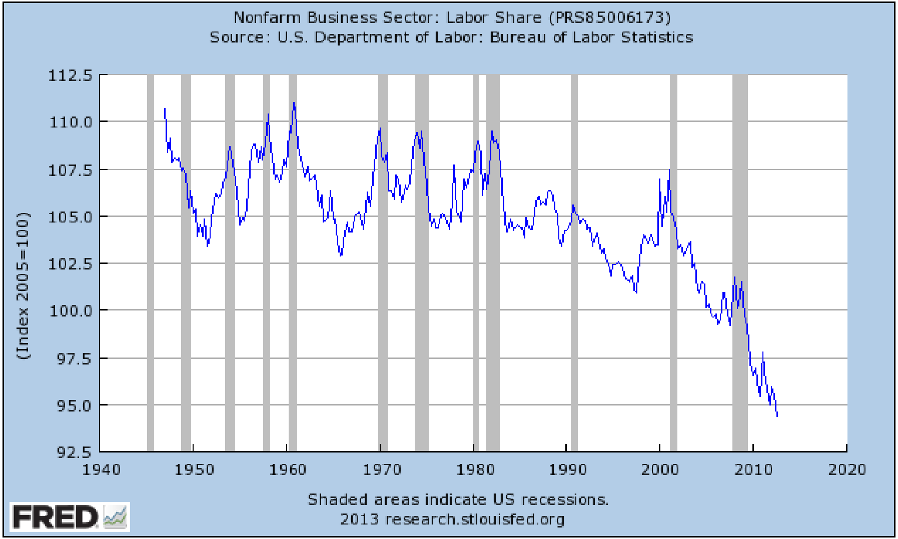
I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action a government could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what exactly? If not, what is it about the nature of India that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else. [Lucas 1988, p. 5]

Stylized facts - Kaldor's facts

Kaldor (1959) popularized facts concerning long run economic growth. Here are some of them:

1. Output per capita, $y = Y/L$ and capital per worker $k = K/L$, grows at constant rates
2. The interest rate is fairly constant over time
3. The output to capital ratio, Y/K, is fairly constant over time
4. The share of value added going to labor and capital are fairly constant

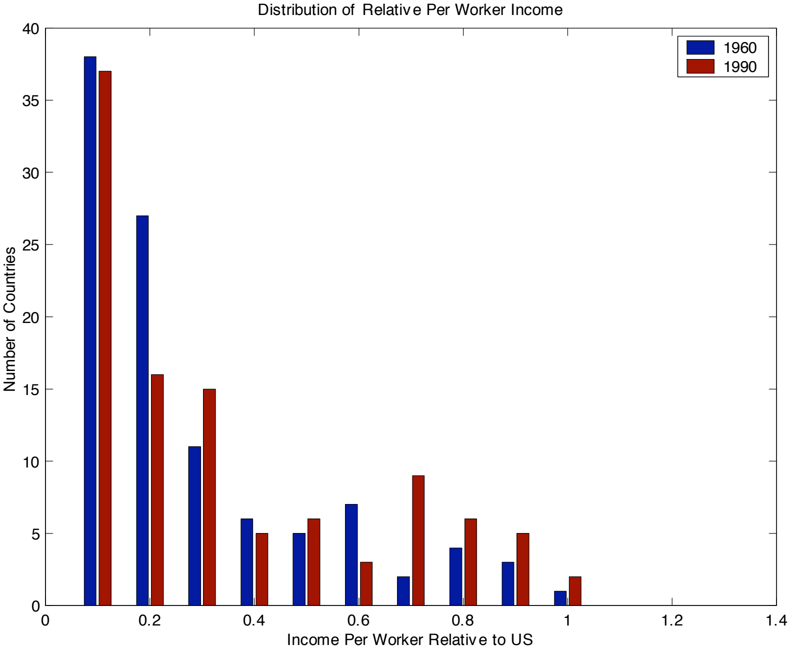
Main source for these slides: Dirk Krueger's notes


Nonfarm Business Sector: Labor Share (PRS85006173)
Source: U.S. Department of Labor: Bureau of Labor Statistics
(Index 2005=100)
112.5
110.0
107.5
105.0
102.5
100.0
97.5
95.0
92.5
1940
1950
1960
1970
1980
1990
2000
2010
2020
Shaded areas indicate US recessions.
2013 research.stlouisfed.org
FRED

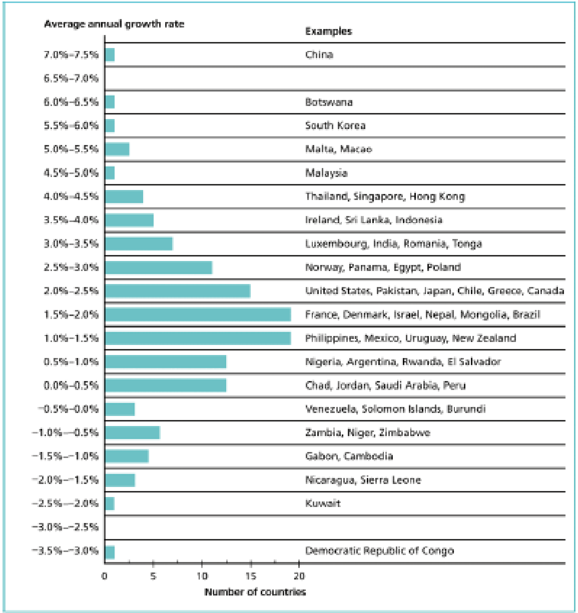
Stylized facts - Summers and Heston data set

Summers and Heston data set:

1. Enormous variation of per capita income across countries. The poorest have 5% of per capita income in the US.
2. Enormous variation in growth rate of per capital income across countries.
3. Growth rate determines economic fate of a country over longer periods of time. How long does it take for a country to double its per capita GDP if it grows at g% per year? About $70/g$. (Lucas, 1988)
4. Countries change their *relative* position in the income distribution.



Distribution of Relative Per Worker Income
1960
1990
Number of Countries
Income Per Worker Relative to US
0
5
10
15
20
25
30
35
40
0
0.2
0.4
0.6
0.8
1
1.2
1.4



Sources: Heston, Summers, and Aten (2006), World Bank (2007a).

Solow Model (1956)

Some preliminary assumptions

- Single good and no international trade
- Factors of production (labor and capital) are fully employed
- Labor force grows at rate $n > 0$ so that (with $L(0) = 1$):

$$L(t) = e^{nt} L(0) = e^{nt}$$

Note that

$$\frac{\dot{L}}{L} \equiv \frac{1}{L}\frac{dL}{dt} = n$$

so that over one unit of time, $dt = 1$, we have $L(t+1) = (1+n)L(t)$

Technology

Technology follows $Y(t) = F(K(t), A(t)L(t))$ with

- F has constant returns to scale, strictly concave, strictly increasing, twice continuously differentiable, $F(0, \cdot) = F(\cdot, 0) = 0$ with Inada.
- Assume labor augmenting technological progress grows at rate $g > 0$:

$$A(t) = e^{gt}$$

- We define

$$\xi(t) \equiv \frac{Y(t)}{A(t)L(t)} = \frac{F(K(t), A(t)L(t))}{A(t)L(t)} = F\left(\frac{K(t)}{A(t)L(t)}, 1\right) \equiv f(\kappa(t))$$

where

$$\kappa(t) \equiv \frac{K(t)}{A(t)L(t)}$$

Capital accumulation

Capital accumulation:

$$\dot{K}(t) = sY(t) - \delta K(t)$$

where $0 < s < 1$ is the exogenous saving rate and $0 < \delta < 1$ is the depreciation rate.

Resource constraint:

$$\underbrace{\dot{K}(t) + \delta K(t)}_{\text{investment}} = Y(t) - C(t)$$

We are basically done with the model. Note:

- No optimizing agents. Behavioral assumption on s.
- Technology grows exogenously

Solving the Solow model

From the capital accumulation equation

$$\frac{\dot{K}}{AL} = s\xi - \delta\kappa$$

Also

$$\frac{\dot{K}}{AL} = \frac{\dot{K}}{K}\frac{K}{AL} = \frac{\dot{K}}{K}\kappa$$

But

$$\frac{\dot{\kappa}}{\kappa} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} - \frac{\dot{A}}{A} = \frac{\dot{K}}{K} - n - g$$

so that

$$\frac{\dot{K}}{AL} = \frac{\dot{K}}{K}\kappa = \left(\frac{\dot{\kappa}}{\kappa} + n + g\right)\kappa$$

$$\dot{\kappa} + \kappa(n+g) = s\xi - \delta\kappa$$

Solving the Solow model

We have found the main equation of the Solow model. The capital accumulation equation in per-effective worker terms:

$$\boxed{\dot{\kappa} = sf(\kappa) - (n + g + \delta)\kappa}$$

It is a first-order nonlinear ordinary differential equation and it completely characterizes our economy for any initial condition $\kappa(0) = K(0)$. Once we have the solution $\kappa(t)_{t\in[0,\infty)}$ we can solve the rest of the model:

$$\begin{aligned}
k(t) &= \kappa(t)A(t) = e^{gt}\kappa(t) \\
K(t) &= e^{(n+g)t}\kappa(t) \\
y(t) &= e^{gt}f(\kappa(t)) \\
Y(t) &= e^{(n+g)t}f(\kappa(t)) \\
C(t) &= (1-s)e^{(n+g)t}f(\kappa(t)) \\
c(t) &= (1-s)e^{gt}f(\kappa(t))
\end{aligned}$$

Solving the Solow model: An Example

Suppose that the production function is Cobb-Douglas: $f(\kappa) = \kappa^\alpha$. The equation:

$$\dot{\kappa} = s\kappa^\alpha - (n + g + \delta)\kappa$$

There are 2 steady-states ($\dot{\kappa} = 0$): $\kappa^* = 0$ and

$$\boxed{\kappa^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}}$$

To solve the equation we define $v(t) = \kappa(t)^{1-\alpha}$. Then

$$\dot{v} = \frac{(1-\alpha)\dot{\kappa}}{\kappa^\alpha}$$

Dividing both sides by $\kappa^\alpha/(1-\alpha)$:

$$(1-\alpha)\frac{\dot{\kappa}}{\kappa^\alpha} = (1-\alpha)s - (1-\alpha)(n+g+\delta)\kappa^{1-\alpha}$$

So that

$$\dot{v} = (1-\alpha)s - (1-\alpha)(n+g+\delta)v$$

Solving the Solow model: An Example

$$\dot{v} = (1-\alpha)s - (1-\alpha)(n+g+\delta)v$$

The general solution to the homogeneous equation is

$$v_g(t) = Ce^{-(1-\alpha)(n+g+\delta)t}$$

for some constant C. A particular equation to the nonhomogeneous equation is

$$v_p(t) = \frac{s}{n+g+\delta} \equiv v^* = (\kappa^*)^{1-\alpha}$$

Therefore, all solutions to this equation are of the form

$$\begin{aligned} v(t) &= v_g(t) + v_p(t) \\ &= v^* + Ce^{-(1-\alpha)(n+g+\delta)t} \end{aligned}$$

Using the initial condition $v(0) = \kappa(0)^{1-\alpha}$:

$$v(t) = v^* + (v(0) - v^*)e^{-(1-\alpha)(n+g+\delta)t}$$

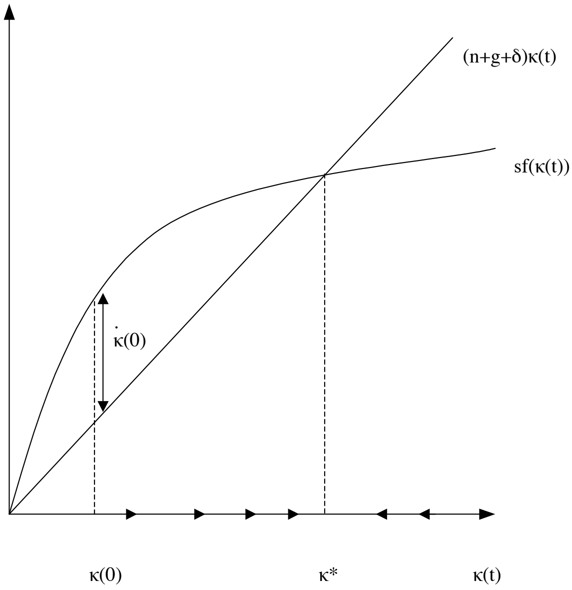
Solving the Solow model: An Example

Going back to our original notation

$$\kappa = \left[\frac{s}{n+g+\delta} + \left(\kappa(0)^{1-\alpha} - \frac{s}{n+g+\delta} \right) e^{-(1-\alpha)(n+g+\delta)t} \right]^{\frac{1}{1-\alpha}}$$

Notice that we would converge to our steady-state regardless of the starting point of the economy.

What to do if we do not have Cobb-Douglas? Graphical analysis.



$(n+g+\delta)\kappa(t)$
$sf(\kappa(t))$
$\dot{\kappa}(0)$
$\kappa(0)$
κ^*
$\kappa(t)$

Empirical evaluation of the Solow model

Does the Solow model fit the data well? Let's see.

If the wage is per unit of labor L we find:

$$
\begin{aligned}
r(t) &= F_K\left(\frac{K(t)}{A(t)L(t)}, 1\right) = F_K\left(\kappa(t), 1\right) \\
w(t) &= A(t)F_L\left(\frac{K(t)}{A(t)L(t)}, 1\right) = A(t)F_L\left(\kappa(t), 1\right)
\end{aligned}
$$

At the balanced growth path (steady state in κ), r is constant and w grows at the same rate as technology.

The capital share is given by $r(t)K(t)/Y(t)$ is constant since K and Y grow at the same rate $n + g$.

The balanced growth path of the Solow model reproduces the four Kaldor facts we've seen. Solow won the Nobel prize in 1989.

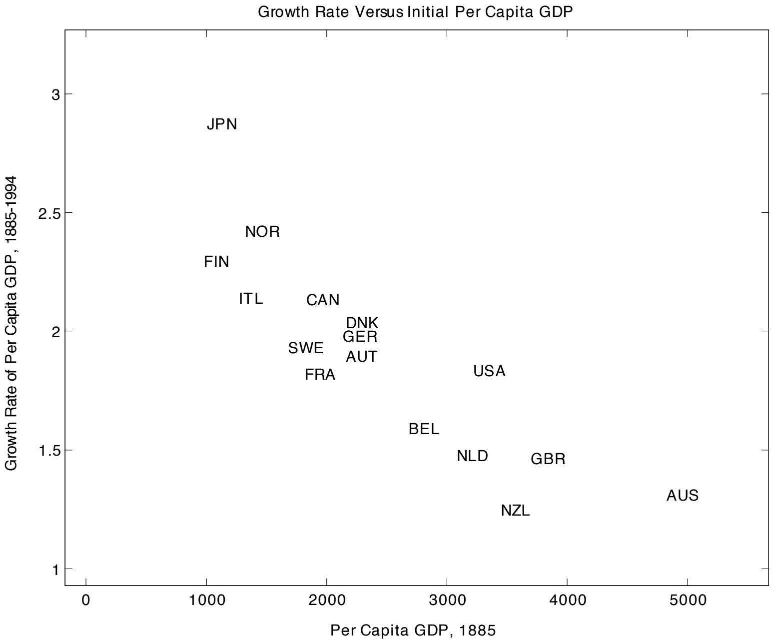
Empirical evaluation of the Solow model

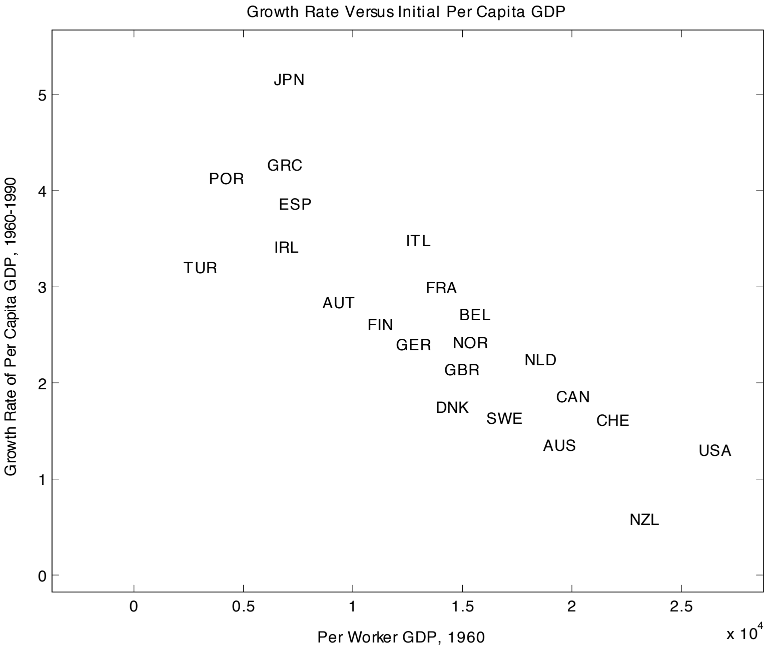
What about the Summers-Heston facts? Can we explain the large difference in per-capita income levels across countries?

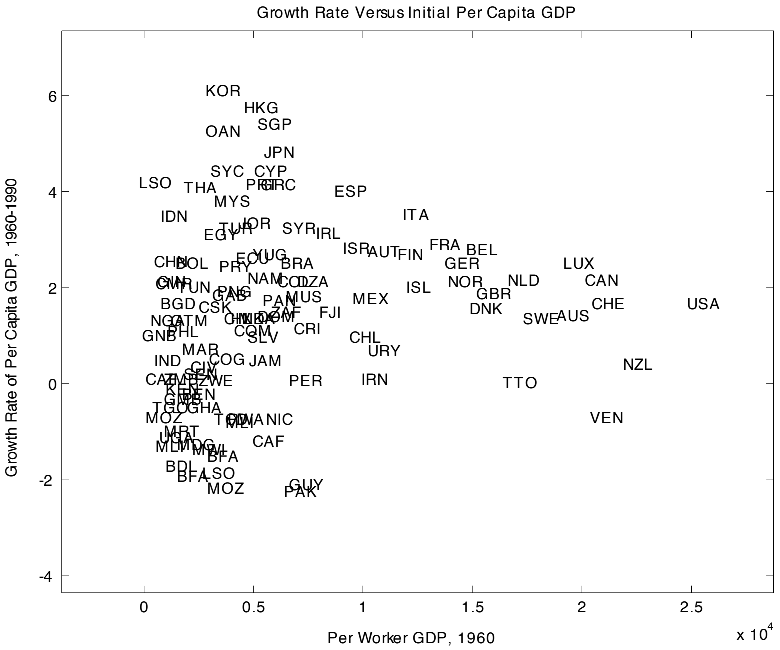
- Suppose all countries have same technology, same population growth and same saving rate
- Then they all converge to same steady-state and per-capita income converges to $y(t) = A(t)f(\kappa^*)$
- So if we observe y to be different across countries some of them have not converged yet
- This implies that poorer countries should be growing faster than rich countries:

$$\begin{aligned}\gamma_y(t) = \frac{\dot{y}(t)}{y(t)} &= g + \frac{f'(\kappa(t))\dot{\kappa}(t)}{f(\kappa(t))} \\ &= g + \frac{f'(\kappa(t))}{f(\kappa(t))}\left(sf(\kappa(t)) - (n+\delta+g)\kappa(t)\right)\end{aligned}$$

Since $\frac{f'(\kappa)}{f(\kappa)}$ and $sf(\kappa) - (n+\delta+g)\kappa$ are decreasing in κ, countries further away from balanced-growth path should grow faster. Data?


Growth Rate Versus Initial Per Capita GDP
Growth Rate of Per Capita GDP, 1885-1994
Per Capita GDP, 1885
3
2.5
2
1.5
1
0
1000
2000
3000
4000
5000
JPN
NOR
FIN
ITL
CAN
DNK
GER
SWE
AUT
FRA
USA
BEL
NLD
GBR
AUS
NZL


Growth Rate Versus Initial Per Capita GDP
Growth Rate of Per Capita GDP, 1960-1990
Per Worker GDP, 1960
x 10^4
0
1
2
3
4
5
0
0.5
1
1.5
2
2.5
JPN
GRC
POR
ESP
ITL
IRL
TUR
FRA
AUT
BEL
FIN
GER
NOR
NLD
GBR
CAN
DNK
SWE
CHE
AUS
USA
NZL



Growth Rate Versus Initial Per Capita GDP
Growth Rate of Per Capita GDP, 1960-1990
Per Worker GDP, 1960
x 10^4
0
0.5
1
1.5
2
2.5
6
4
2
0
-2
-4
KOR
HKG
SGP
OAN
JPN
SYC
CYP
LSO
THA
ESP
MYS
IDN
ITA
SYR
IRL
EGY
ISR
AUT
FIN
FRA
BEL
GER
LUX
BOL
PRY
BRA
NAM
ISL
NOR
NLD
CAN
TUN
MEX
GBR
MUS
PAN
BGD
CSK
DNK
CHE
USA
FJI
SWE
AUS
CRI
CHL
SLV
MAR
URY
IND
COG
JAM
NZL
PER
IRN
TTO
GHA
MOZ
NIC
VEN
CAF
BFA
BDI
GUY
PAK

Empirical evaluation of the Solow model

This idea that countries should converge to the same path is called absolute convergence. Should we discard the Solow model?

Empirical evaluation of the Solow model

This idea that countries should converge to the same path is called absolute convergence. Should we discard the Solow model?

What if parameters (s, n, δ) are different?

- This idea is called conditional convergence
- Now the fast-growing countries are the ones away from *their own* BGP
- The per-capita incomes should still *grow at the same rate* once countries reach BGP

Empirical evaluation of the Solow model

To test conditional convergence:

- Compute the steady-state output per worker that a country should possess in a given initial period, say 1960, given (s, n, δ) measured in this country's data.
- Then measure actual GDP per worker in this period and build the difference. This difference indicates how far the country is from its BGP.
- Plot the difference against the growth rate of GDP per worker from the initial period to the current period.
- Countries that are further away from their BGP should grow faster.

The data seems to confirm the Solow model. (See Jones 1998 Figure 3.8)

Look at Mankiw, Romer and Weil for a test of the Solow model.

Moving from the Solow model

Several points:

- Constant saving rate is a strong behavioral assumption that is not derived from maximizing agents. Relaxing it leads to the Cass-Koopmans-Ramsey model.
- Technological progress is modeled exogenously. Relaxing this leads to the endogenous growth model.

We now look at the Cass-Koopmans-Ramsey model.

Cass-Koopmans-Ramsey model

We carry over most assumptions from Solow:

- Population grows at rate $n > 0$
- Production function $Y = F(K, AL)$
- Technology A grows at rate $g > 0$

Aggregate capital stock evolves according to

$$\dot{K} = F(K, AL) - \delta K - C$$

Notice that we have consumption here and no exogenous saving rate.

We can write the equation in per-effective labor terms:

$$\dot{\kappa} = f(\kappa) - \zeta - (n + \delta + g)\kappa$$

where $\zeta = C/(AL)$ and $\kappa = K/(AL)$.

Cass-Koopmans-Ramsey model

We have a representative agent with utility ($\rho > 0$ is discount factor):

$$u(c) = \int_0^\infty e^{-\rho t} \underbrace{\frac{(c(t))^{1-\sigma}}{1-\sigma}}_{U(c)} dt$$

By using our other notation:

$$\begin{aligned} e^{-\rho t} U(c(t)) &= e^{-\rho t} \frac{(c(t))^{1-\sigma}}{1-\sigma} \\ &= e^{-\rho t} \frac{(\zeta(t) e^{gt})^{1-\sigma}}{1-\sigma} = e^{-(\rho - g(1-\sigma))t} \frac{(\zeta(t))^{1-\sigma}}{1-\sigma} \end{aligned}$$

We assume $\rho > g(1-\sigma)$ and define $\hat{\rho} = \rho - g(1-\sigma)$:

$$u(c) = \int_0^\infty e^{-\hat{\rho} t} U(\zeta(t)) dt = \int_0^\infty e^{-\hat{\rho} t} \frac{(\zeta(t))^{1-\sigma}}{1-\sigma} dt$$

Cass-Koopmans-Ramsey model

The Social Planner problem is

$$\max_{(\kappa,\zeta)\geq 0} \int_0^\infty e^{-\hat{\rho}t} U(\zeta(t))dt$$

$$\text{s.t. } \dot{\kappa}(t) = f(\kappa(t)) - \zeta(t) - (n+\delta+g)\kappa(t)$$

$$\kappa(0) = \kappa_0$$

This problem can be solved using Pontryagin's maximum principle.

- $\kappa(t)$ is the state variable
- $\zeta(t)$ is the control
- $\lambda(t)$ denotes the co-state variable associated with $\kappa(t)$

One reference is Intriligator (2002) *Mathematical Optimization and Economic Theory*.
We will look more at how to solve these problems when discussing search models.

Cass-Koopmans-Ramsey model

Write the Hamiltonian

$$\mathcal{H}(t,\kappa,\zeta,\lambda) = e^{-\hat{\rho}t}U(\zeta(t)) + \lambda(t)\left[f(\kappa(t)) - \zeta(t) - (n+\delta+g)\kappa(t)\right]$$

Sufficient conditions for a solution are

$$\frac{\partial\mathcal{H}(t,\kappa,\zeta,\lambda)}{\partial\zeta(t)} = 0$$

$$\dot{\lambda}(t) = -\frac{\partial\mathcal{H}(t,\kappa,\zeta,\lambda)}{\partial\kappa(t)}$$

$$\lim_{t\to\infty}\lambda(t)\kappa(t) = 0$$

Plugging in we find the four equations:

$$e^{-\hat{\rho}t}U'(\zeta(t)) = \lambda(t)$$

$$\dot{\lambda}(t) = -(f'(\kappa(t)) - (n+\delta+g))\lambda(t)$$

$$\lim_{t\to\infty}\lambda(t)\kappa(t) = 0$$

$$\dot{\kappa}(t) = f(\kappa(t)) - \zeta(t) - (n+\delta+g)\kappa(t)$$

Cass-Koopmans-Ramsey model

Differentiating the first equation w.r. to time and combining it with itself

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\zeta} U''(\zeta)}{U'(\zeta)} - \hat{\rho}$$

Combining with the second equation:

$$\dot{\zeta}\frac{\zeta U''(\zeta)}{U'(\zeta)} = -\zeta \left(f'(\kappa) - (n + \delta + g + \hat{\rho}) \right)$$

Because of CRRA the first fraction on the LHS is equal to $-\sigma$:

$$\boxed{\dot{\zeta}(t) = \frac{1}{\sigma}\zeta(t) \left(f'(\kappa(t)) - (n + \delta + g + \hat{\rho}) \right)}$$

Cass-Koopmans-Ramsey model

Therefore, any allocation (κ, ζ) that satisfies the following system is PO:

$$\dot{\zeta}(t) = \frac{1}{\sigma}\zeta(t)\left(f'(\kappa(t)) - (n + \delta + g + \hat{\rho})\right)$$
$$\dot{\kappa}(t) = f(\kappa(t)) - \zeta(t) - (n + \delta + g)\kappa(t)$$

with $\kappa(0) = \kappa_0$ and the Transversality Condition.

We have a steady-state ($\dot{\zeta} = \dot{\kappa} = 0$) at

$$\boxed{f'(\kappa^*) = (n + \delta + g + \hat{\rho})}$$

$$\boxed{\zeta^* = f(\kappa^*) - (n + \delta + g)\kappa^*}$$

Cass-Koopmans-Ramsey model

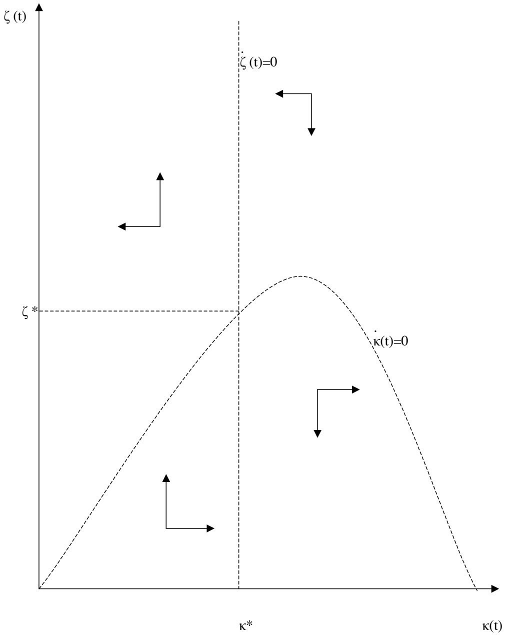
How to study the economy out of steady-state? Phase diagram.

$$\dot{\zeta}(t) = \frac{1}{\sigma}\zeta(t)\left(f'(\kappa(t)) - (n + \delta + g + \hat{\rho})\right)$$
$$\dot{\kappa}(t) = f(\kappa(t)) - \zeta(t) - (n + \delta + g)\kappa(t)$$

We first find the two isocline:

$$\dot{\zeta} = 0 \Rightarrow f'(\kappa(t)) - (n + \delta + g + \hat{\rho}) = 0$$
$$\dot{\kappa} = 0 \Rightarrow f(\kappa(t)) - \zeta(t) - (n + \delta + g)\kappa(t) = 0$$

In the (κ, ζ) plan, the first one is a vertical line at $\kappa =$ constant. The second one is strictly concave in κ.



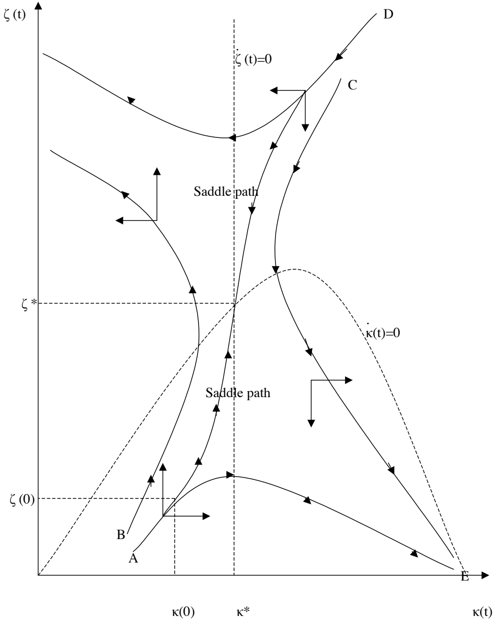
ζ (t)
ζ̇ (t)=0
ζ *
κ̇(t)=0
κ*
κ(t)

Cass-Koopmans-Ramsey model

We have an initial condition $\kappa(0) = \kappa_0$. We need in general two conditions to pin down a path for the system. The question is how should the planner pick $\zeta(0)$.

We argue two things

1. For a given $\kappa(0) > 0$ any choice $\zeta(0)$ of the planner leading to a path not converging to the steady state (κ^*, ζ^*) cannot be optimal
2. There is a unique stable path leading to the steady state. This is called saddle-path stability of the steady state and the unique stable path is called a saddle path.



ζ (t)
D
$\dot{\zeta}$ (t)=0
C
Saddle path
ζ *
$\dot{\kappa}$(t)=0
Saddle path
ζ (0)
B
A
E
κ(0)
κ*
κ(t)

Cass-Koopmans-Ramsey model

For point 1

- Paths going toward point E lead to negative consumption in finite time
- Paths converging to north-west hit $\zeta = 0$ which means $\kappa = 0$ forever. Because of Inada, this cannot be optimal.

For point 2

- We would need to study the dynamic around the steady state by using local approximation (we skip that part here, but it is true)

Combining results

- The only paths converging to the unique steady state can be optimal solutions
- Locally, around the steady state, the converging path is unique (the saddle path)
- Given an initial κ_0 the planner picks the $\zeta(0)$ that puts the economy on the saddle path.

Cass-Koopmans-Ramsey model

What is the behavior of the economy once it reached the steady state? Per capita variables grow at rate g and aggregate variable grow at rate n:

$$
\begin{aligned}
c(t) &= e^{gt}\zeta^* \\
k(t) &= e^{gt}\kappa^* \\
y(t) &= e^{gt}f(\kappa^*) \\
C(t) &= e^{(g+n)t}\zeta^* \\
K(t) &= e^{(g+n)t}\kappa^* \\
Y(t) &= e^{(g+n)t}f(\kappa^*)
\end{aligned}
$$

The long-run behavior of this model is identical to that of the Solow model

- The economy converges to a balanced growth path
- We can understand the Cass-Koopmans-Ramsey model as a micro foundation for the Solow model

Endogenous Growth Models

Second issue with Solow model was the exogenous growth in technology. Here we move away from this with the basic AK model:

- No technological progress. Production function $Y(t) = AK(t)$ with A fixed.
- Population grows at rate $n > 0$
- Preference of representative agent given by

$$U(c) = \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} dt$$

- Budget constraint

$$c(t) + \dot{a}(t) + na(t) = w(t) + (r(t) - \delta)a(t)$$

  where $a = A/L$ is per capita asset holdings and with $a(0) = k_0$ given. We would also need a No-Ponzi scheme condition.
- Firm problem

$$\max_{K(t),L(t)\geq 0} AK(t) - r(t)K(t) - w(t)L(t)$$

AK Model

Definition 1

A sequential markets equilibrium are allocations for the household $(c(t), a(t))_{t\in[0,\infty)}$, allocations for the firm $K(t), L(t))_{t\in[0,\infty)}$ and prices $(r(t), w(t))_{t\in[0,\infty)}$ such that

1. Given prices, the allocation $(c(t), a(t))_{t\in[0,\infty)}$ solves the household's problem.
2. Given prices, the allocation $(K(t), L(t))_{t\in[0,\infty)}$ solves the firm's problem.
3. $L(t) = e^{nt}$, $L(t)a(t) = K(t)$, $L(t)c(t) + \dot{K}(t) + \delta K(t) = AK(t)$

AK Model

We can solve the household's problem like Cass-Koopman. Solution:

$$\dot{c} = \frac{1}{\sigma}(r - (n + \delta + \rho)c$$

$$\gamma_c = \frac{\dot{c}}{c} = \frac{1}{\sigma}(r - (n + \delta + \rho))$$

Since $r(t) = A$ (remember that it is not changing), we can write

$$\gamma_c = \frac{\dot{c}}{c} = \frac{1}{\sigma}(A - (n + \delta + \rho))$$

Consumption per capital is always growing at the same rate (not just at the steady-state). Integrating, we find

$$\boxed{c(t) = c(0)e^{\frac{1}{\sigma}(A-(n+\delta+\rho))t}}$$

We make the following assumptions on the parameters

- $A - (n + \delta + \rho) > 0$ so that we have positive growth.
- $\frac{1-\sigma}{\sigma}\left[A - (n + \delta) - \frac{\rho}{1-\sigma}\right] \equiv \phi < 0$ so that the integral of discounted consumption is finite.

AK Model

What is the behavior of capital per capita? From resource constraint:

$$\dot{K}(t) + \delta K(t) + C(t) = AK(t)$$
$$c(t) + \dot{k}(t) = Ak(t) - (n + \delta)k(t)$$

So that

$$\gamma_k(t) = \frac{\dot{k}(t)}{k(t)} = A - (n + \delta) - \frac{c(t)}{k(t)}$$

In a *balanced-growth path* γ_k is constant so that c and k must grow at the same rate

$$\boxed{\gamma_k = \gamma_c = A - (n + \delta + \rho)}$$

AK Model

What if we are *not* at the balanced-growth path?

$$\dot{k}(t) = -c(0)e^{\frac{1}{\sigma}(A-(n+\delta+\rho))t} + Ak(t) - (n+\delta)k(t)$$

Exercise: Solve this equation! (Use general + particular solution and use transversality to pin down constant).

The solution is

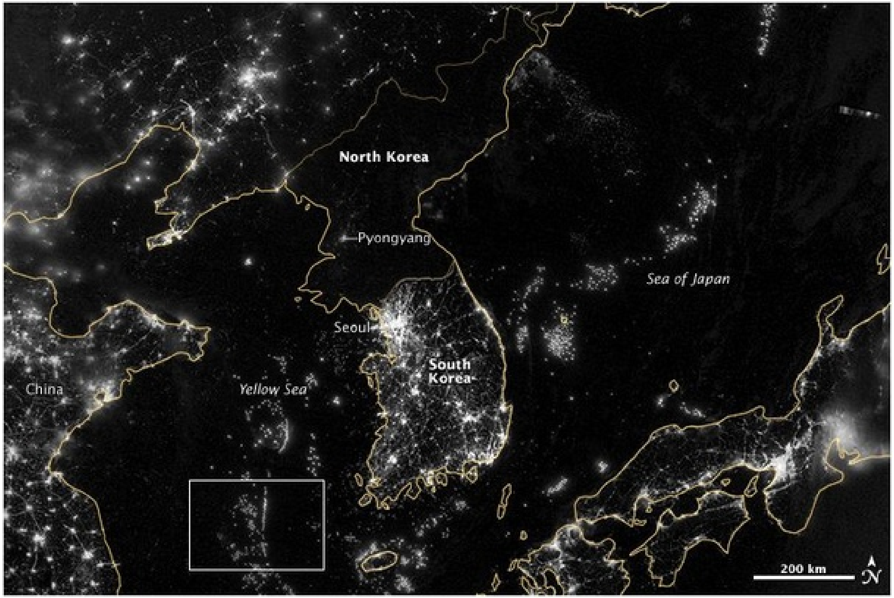
$$k(t) = -\frac{c(0}{\phi}e^{\frac{1}{\sigma}(A-(n+\delta+\rho))t} = -\frac{c(t)}{\phi}$$

- The capital stock is always proportional to consumption.
- Since consumption always grows at a constant rate, so does capital.
- The initial condition $k(0) = k_0$ pins down $c(0) = -\phi k(0)$ and $y(0) = Ak(0)$.
- All variables grow at the same rate $\gamma_c = \gamma_k = \gamma_y$.

AK Model

Why do we care about this model?

- In Solow and Cass-Koopman model, the growth rate of the economy was $\gamma_c = \gamma_k = \gamma_y = g$, the same as growth in technology.
- In Solow and Cass-Koopman model, parameters like s, n, δ affect per-capita income *level* but not *growth rate*.
- In AK model, the growth rate is determined by all these parameters!
- No convergence in AK model
- Key piece for this result: absence of decreasing returns to capital


North Korea
Pyongyang
Seoul
South Korea
China
Yellow Sea
Sea of Japan
200 km
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