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Problem Set 3
Macroeconomics I
Due November 26

Recall the single-sector neoclassical model with search from class. In the first problem set, we solved this model using a linear approximation to the model around its non-stochastic steady state. In the second problem set, we solved the model using a shooting algorithm.

Now, we are going to solve the non-linear using a value function iteration over a discretized state space. You should feel free to build on the example code shared on Canvas.

The parameters you should use in this problem set are listed below, they are the same as in Problem Set 1 and 2.

Table 1: Numerical Parameter Values

Concept	Symbol	Value
Discount factor	β	0.99
Inverse IES	σ	2.00
Capital share	α	0.30
Capital Depreciation	δ_k	0.03
Labor separation	δ_n	0.10
Vacancy cost	ϕ_n	0.50
Matching Function Level	χ	1.00
Matching Function Elasticity	ε	0.25
log(A) persistence	ρ	0.95
log(A) disturbance	σ_a	0.01

1. **Value Function iteration:**

   (a) Generate a script called `solve_vf`. The first part of the script should use our tools to generate the solution matrices g_x and h_x to the linearized (NOT log-linearized) model at the baseline parameters. Update your model solution to include the (approximated) value function as an additional element of the Y vector.

   (b) Use the command `AR1_rouwen` to create a grid of 5 points for the log of productivity, centered around $log(A) = 0$. Exponentiate this grid, to create a 5 point grid on the level of productivity. Using the Markov transition matrix θ produced by `AR1_rouwen`, show (numerically) that the expected value $E[A_t] = 1.0005$. Briefly explain why this number is greater than one.

(c) Using the `linspace` command, create a grid of 50 points for capital. The grid should be equally-spaced between $0.9 \times \bar{K}$ and $1.1 \times \bar{K}$. Similarly, create a grid of 150 points for employment. The grid should be equally-spaced between $0.8 \times \bar{N}$ and $1.2 \times \bar{N}$.

(d) Now, use the command `ndgrid` to create a grid across all $5 \times 50 \times 150$ points in the state-space. Call the vector of values for productivity `agrid`, call the vector of values for capital `kgrid`, call the vector of value for employment `ngrid` for each point in the state-space.

(e) Using the linearized policy functions g_x and h_x and the grid you generated in part (c) above, compute initial guess for the policy choices `kinit`, `ninit`, and the value function `vinit`.

(f) For each of 50×150 points in the policy-space, use your guess of `vinit` to compute (our current best estimate of) $E[V(K_{t+1}, N_t, A_{t+1}|A_t)]$ for possible values of the current exogenous state A_t. For example, when A_t is at the lowest value in the grid, and $\{K_{t+1}, N_{t+1}\}$ are also at their lowest value,

$$E[V(K_{t+1}, N_t, A_{t+1}|A_t)] = -3.65X72129X050267.$$

Report the missing digits, marked with an X, from above.

(g) For each of $5 \times 50 \times 150$ points in the state-space, compute the approximate value function for all 50×150 given each candidate policy choices $\{K_{t+1}, N_t\}$ using the period utility function and our (estimate) of $E[V(K_{t+1}, N_t, A_{t+1}|A_t)]$ from above. If a candidate choice implies negative consumption, let the value function at that point be `-infty`.

(h) For each of $5 \times 50 \times 150$ points in the state-space, select the best possible policy choice from the 50×150 candidate policies using Matlab's `max` function. Keep track of the associated policy function and the index of the optimal choice.

(i) Repeat steps 1f to 1h until convergence (use a criterion of 1e-6). This is your baseline policy function iteration. For example, when A_t is at the lowest value in the grid, and $\{K_{t+1}, N_{t+1}\}$ are also at their lowest value, the final policy function

$$V(K_t, N_{t-1}, A_t) = -3.6826.$$

(j) Plot your policy functions for capital and labor choices against the values of the capital in the grid. Assume that TFP is held constant at 1. And that N_{t-1} is at the 75th value on its grid. Use a blue line for the initial linear policy functions and a green line for the policy functions you solved for in the non-linear problem. What do you notice about the discretized solution?

(k) *Simulate 5000 periods the economy using the policy functions from the value function iteration. Fill in the table below.

Table 2: Moments from simulated linear model

Moment	linear model value	value function model value
Std $\log(Y)$	0.0522	
Std $\log(C)$		
Std $\log(I)$	0.0975	
Std $\log(N)$		