ECON 6090-Microeconomic Theory. TA Section 1

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December 6, 2024

In Section notes

Preference

$$\succsim \text{ rational } \iff \succsim \text{ complete and transitive}$$

Indifference relation

$$x \sim y \iff x \succsim y \text{ and } y \succsim x$$

Strictly preferred relation

$$x \succ y \iff x \succsim y \text{ and } \neg[y \succsim x]$$

1. From preference we have $C^*(B, \succsim)$

$$\succsim \text{ is rational } \iff C^*(B, \succsim) \text{ satisfies HWARP for } B \in P(x)$$
$$\iff \text{ Sen's } \alpha, \beta$$

2. From choice structure

$$(\mathcal{B}, C(.)) \implies \succsim^* \text{ revealed preference}$$

   If

   (a) $(\mathcal{B}, C(.))$ satisfies WARP

   (b) $\mathcal{B}$ is the power set of X[1]

   Then we have that $\succsim^*$ is rational

Exercises

Rational Preference Relations

1. Yes.

$$\forall x, y, z \in X$$
$$x \sim y, y \sim z$$
$$\implies x \succsim y, y \succsim x \text{ and } y \succsim z, z \succsim y$$
$$\implies x \succsim y \succsim z \text{ and by transitivity } z \succsim y \succsim x$$
$$\implies x \sim z$$

2. (a) Since $x \succsim x \implies x \geq x + \epsilon$, which is a contradiction, it is not complete, therefore not rational.

   (b) Let $y = x - \epsilon, z = x + \epsilon$

$$\implies y \succsim x, x \succsim z$$

   By transitivity,

$$y \succsim z$$

   Which is a contradiction. Therefore, not rational.

[1] We can weaken the claim by $\mathcal{B}$ only being all subsets of X up to 3 elements

(c) Rational.

$$\succsim: c \succ b \succ a$$

(d) We check if reflexivity holds,

$$x \succsim_2 x \implies x \succ_1 x$$

Which is a contradiction. Not complete. Not rational.

(e) Let $(x_1, x_2) \succsim (y_1, y_2)$ and $(y_1, y_2) \succsim (z_1, z_2)$.We do an analysis by cases:

i. If $x_1 > y_1$ and $[(y_1 > z_1)$ or $(y1 = z_1$ and $y_2 \geq z_2)]$ we have $(x_1, x_2) \succsim (z_1, z_2)$

ii. If $x_1 = y_1$ and $x_2 \geq y_2$ and $[(y_1 > z_1)$ or $(y_1 = z_1$ and $y_2 \geq z_2)]$ we have $(x_1, x_2) \succsim (z_1, z_2)$.

So it is transitive.
Now to prove completeness we check if $(x_1, x_2) \succsim (y_1, y_2)$ or $(y_1, y_2) \succsim (x_1, x_2)$ or both hold.

i. If $x_1 = y_1$ either $x_2 \geq y_2$ or $x_2 \leq y_2$

ii. If $x_1 \neq y_1$ either $x_1 > y_1$ or $x_1 < y_1$

So it is complete.

3. (2022 Q)

(a) We proceed by induction.

i. Let $A^1 = \{a_1\}$. Then a_1 is the best alternative (BA).

ii. Let $A^2 = \{a_1, a_2\}$. Then by completeness, either a_1 or a_2 or both are BA.

iii. Now assume that for $A^{N-1} = \{a_1, a_2, ..., a_{N-1}\}$ there exist $a^* \in A^{N-1}$ that is BA. Then for $A^N = a_1, a_2, ..., a_N$, we have

1) $a^* \succsim a_N$, and then a^* is BA in A^N

2) $a_N \succsim a^*$, and then by transitivity $a_N \succsim a^* \succsim a_j$ for all $j = 1, ..., N-1$

$$\implies a_N \text{ is BA}$$

(b) By definition of BA, $a' \in A' \subseteq A$. Again by definition of BA, $a^* \succsim a \ \forall a \in A$ which implies that $a^* \succsim a'$.

Choice Rules

1. Observe that

$$C^*(\{a, b, c\}, \succsim) = \{a, b\} \implies a \sim b$$

$$C^*(\{a, b, c\}, \succsim) = \{b\} \implies b \succ a$$

And we get a contradiction. Not rational.

2. There is no rational preference relation consistent with the information given about $C(.)$. Observe that,

$$C^*(\{a, b, c\}, \succsim) = a, b \implies a \sim b \succ c$$

$$C^*(\{b, c\}, \succsim) = \{b\} \implies b \succ c$$

$$C^*(\{c, d\}, \succsim) = \{c\} \implies c \succ d$$

$$C^*(\{a, d\}, \succsim) = \{a, d\} \implies a \sim d$$

Combining this information,

$$\implies a \sim b \succ c \succ d$$

$$\implies a \succ d$$

Which is a contradiction.