ECON 6090 - TA Section 4

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Exercises

Consumer Theory

1. (2009 Prelim 1) There are three goods with quantities denoted by $x = (x_1, x_2, x_3) \in \mathbb{R}^3_+$. A consumer's preferences can be represented by the utility function $u(x) = x_1 x_2^{1/2} x_3^{1/2}$. The prices of the goods are represented by $p \in \mathbb{R}^3_{++}$ and the consumer has wealth $w > 0$.

   (a) Write the consumer's decision problem as a constrained optimization problem.

   (b) Find the consumer's demand functions for the three goods.

   Now suppose that in addition to using money to purchase goods the consumer also has to provide coupons in order to make a purchase. The purchase of $y \geq 0$ units of any good requires y coupons. The consumer has an endowment of $c > 0$ coupons.

   (c) Write the consumer's new decision problem as a constrained optimization problem.

   (d) Is it possible that at a solution to the consumer's problem he has some left-over coupons? That is, can the coupon constraint ever be non-binding?

   (e) Suppose that $p = (1, 1, 1)$. Find the consumer's demands for the three goods.

2. (2023 Prelim 1) A NGO (non-government organization) has a budget (amount of money that it can spend) $T > 0$ that it must allocate across n activities such that $\sum_{i=1}^{n} e_i = T$, where $e_i \geq 0$ is the expenditure on activity i. Let $B(e)$, where $e = (e_1, ..., e_n)$, be the benefit from expenditure e on activities. Assume that $B : R_+^n \rightarrow R^1$ is strictly increasing, strictly concave and smooth. The NGO's objective is to chose e to maximize the benefit obtained from e. Let $V(T)$ be the value of the NGO's decision problem; that is,$V(T)$ is the maximum benefit that can obtained from a budget of T.

   (a) Show that the value of the decision problem is strictly increasing in T . For this part of the problem do not assume that $V(T)$ is differentiable.

   (b) Let $e^*(T)$ be the solution to the decision problem. Assume that the value of the decision problem and the solution are differentiable, and that $e^*(T) >> 0$ for any $T > 0$. We are interested in how the value of the decision problem changes as the budget changes. Compute $\frac{dV(T)}{dT}$. Is this derivative equal to $\frac{\partial B(e^*(T))}{\partial e_i}$? Explain carefully.

   (c) Now suppose that that there are only two activities and that $B(e) = \sum_{i=1}^{2} b_i(e_i)$ where each $b(e_i)$ is strictly increasing, strictly concave and smooth. Suppose that because of some new inefficiency in activity one that the benefit from spending e1 on activity one changes to $b(\alpha e_1)$, where $0 < \alpha < 1$. The benefit functions for the other activities are unchanged. What happens to the optimal choice of e_1? Explain carefully.