Towards Applications of Reference Dependence

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Overview

- In early applications of reference dependence and loss aversion, there were (at least) three major degrees of freedom:
    - assumptions about when experience gain-loss utility
    - assumptions about what is the reference point
    - assumptions about magnitude of gain-loss utility
- Koszegi & Rabin (2006, 2007) tried to impose some discipline, and <u>some</u> later applications were heavily influenced by their approach.
- For now, three early mini-applications:
    - Endowment effect (already done)
    - Aggregate bets
    - Risk aversion

Application: Aggregate Bets

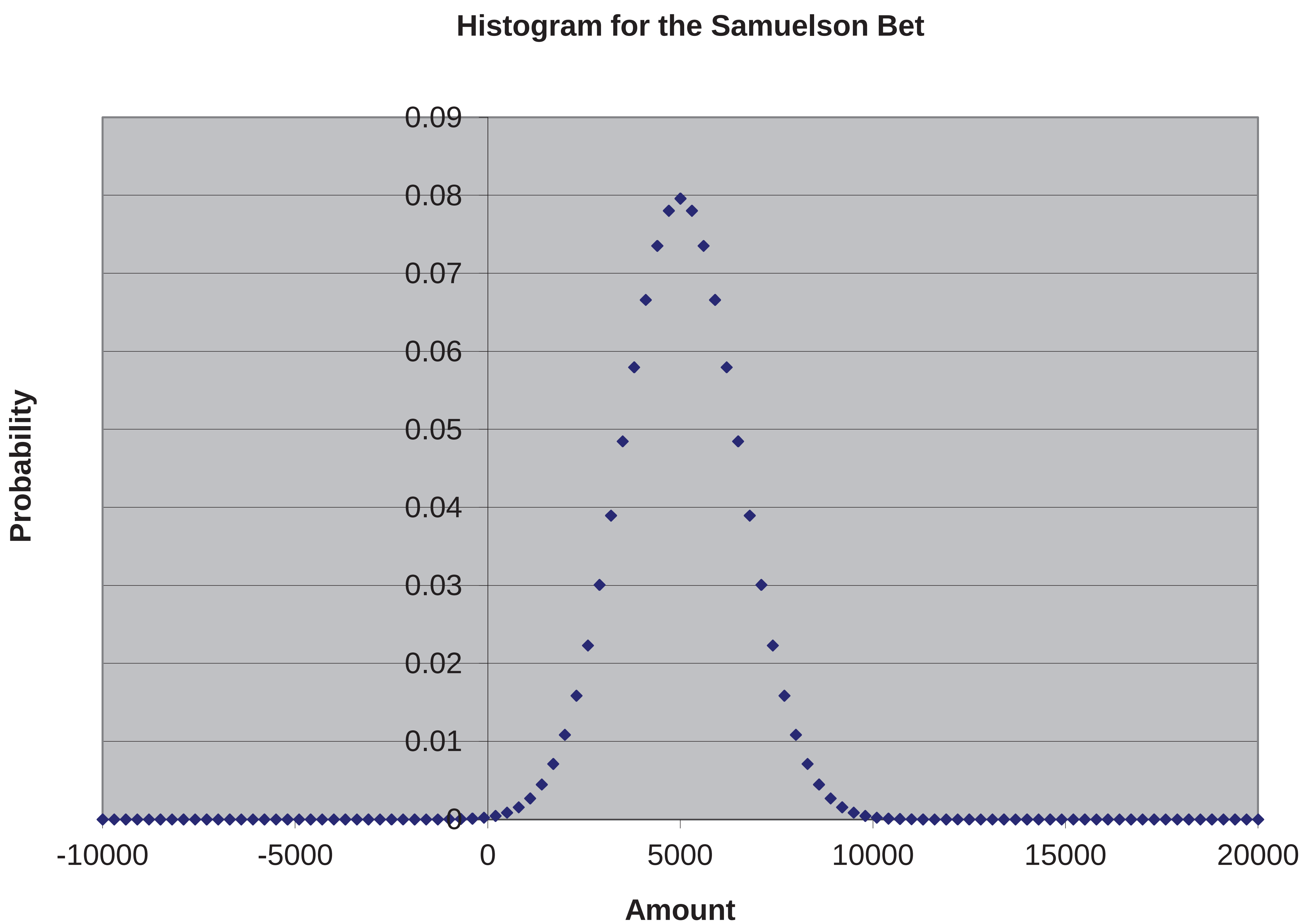
Example due to Samuelson (1963)

Consider the following bet:

$$\begin{aligned} &\text{win \$200 with prob } 1/2 \\ &\text{lose \$100 with prob } 1/2 \end{aligned}$$

Samuelson's colleague turned down this bet, but announced that he would accept 100 independent plays of the same bet.

Samuelson proved that his colleague was "irrational"—by proving that it is inconsistent with EU theory to turn down a single bet but to accept 100 independent plays of that same bet.



Histogram for the Samuelson Bet
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Probability
-10000
-5000
0
5000
10000
15000
20000
Amount

Application: Aggregate Bets

Behavioral economists' interpretation—see in particular Benartzi & Thaler (1995) and Thaler & Rabin (2001):

- It is a feature of humans to both:
  - be averse to 50-50 bets to lose $\$Y$ vs. win $\$X > \$Y$
  - be attracted to multiple independent plays of that same bet

Application: Aggregate Bets

Consider a simple model of loss aversion:

- Suppose that a person evaluates bets according to the value function (and uses $\pi(p) = p$):

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2.5x & \text{if } x \leq 0 \end{cases}$$

Consider whether to accept a single bet $\mathbf{y} = [200, .5; -100, .5]$.

- $E[v(\mathbf{y})] = -25$
  $\Longrightarrow$ don't take the bet.

Application: Aggregate Bets

Consider whether to accept two independent plays of the bet y.

Case 1: Suppose that you evaluate them independently ("watch bets being played out").

- Get total utility $2E[v(\mathbf{y})] = -50$
  $\Longrightarrow$ don't take aggregate bet.

Case 2: Suppose that you evaluate them together ("don't watch bets being played out").

- Then face gamble $\mathbf{z} = [400, .25; 100, .5; -200, .25]$.
- $E[v(\mathbf{z})] = 25$
  $\Longrightarrow$ accept aggregate bet.

Point: Loss aversion can lead a person to reject one play of the bet but accept multiple independent plays of the bet.

Application: Risk Aversion

Rabin (Econometrica 2000)
[follow-up by Rabin & Thaler (JEP 2001)]

"Risk Aversion":

- People tend to dislike risky prospects even when they involve an expected gain.
  - E.g.: A 50-50 gamble of losing $100 vs. gaining $105.

Economists' explanation:

- EU theory with a concave utility function.

Rabin's Point:

- Calibrationwise, this explanation doesn't work, because according to EU theory, "anything but virtual risk neutrality over modest stakes implies manifestly unrealistic risk aversion over large stakes."

Application: Risk Aversion

Arrow's in-the-limit result:

- Consider any gamble $\mathbf{x} \equiv (x_1, p_1; \ldots; x_N, p_N)$ with $E\mathbf{x} > 0$.
- For any scalar $\phi > 0$, let $\phi\mathbf{x} \equiv (\phi x_1, p_1; \ldots; \phi x_N, p_N)$.
- Then for any concave u, there exists $\bar{\phi} > 0$ such that accept $\phi\mathbf{x}$ for all $\phi \in (0, \bar{\phi})$.

Arrow's Point: Under EU, any concave utility will yield approximate risk neutrality for small enough stakes.

Rabin's Point: Under EU, for applied purposes, people must be approximately risk-neutral even for modest stakes.

Application: Risk Aversion

Rabin's result—an example:

- Suppose that Johnny is a "risk-averse" EU maximizer ($u'' \leq 0$).
- Suppose that, for any initial wealth, Johnny will reject a 50-50 gamble of losing \$10 vs. gaining \$11.
- Now consider a 50-50 gamble of losing \$100 vs. gaining $\$X$.
- What is the minimum X such that Johnny might accept?
- Answer:

Application: Risk Aversion

Rabin's result—other examples:

If for any w turn down 50/50 bet of...	then for any w turn down 50/50 bet of...
lose \$10 / gain \$11	lose \$100 / gain ∞
lose \$10 / gain \$10.10	lose \$1000 / gain ∞
lose \$100 / gain \$105	lose \$2000 / gain ∞
lose \$1000 / gain \$1050	lose \$20,000 / gain ∞

Rabin's result—a modified example:

Modified version:

- Suppose the person rejects a 50-50 gamble of lose \$10/win \$11 for all wealth levels less than \$300,000.
- Then for initial wealth \$290,000, she'll reject a 50-50 gamble of lose \$100/win $\$X$ for all $\$X < \$71,819$.

If for all $w < \$300k$ turn down 50/50 bet of...	then for $w = \$290k$ turn down 50/50 bet of...
lose \$100 / gain \$105	lose \$2000 / gain \$69,900
lose \$1000 / gain \$1050	lose \$20,000 / gain \$690,900

Application: Risk Aversion

An alternative framing

Consider the following preferences over 50-50 bets to lose Y vs. gain X:

- For $Y = 10$, accept if and only if $X > 12$.
- For $Y = 100$, accept if and only if $X > 120$.
- For $Y = 1000$, accept if and only if $X > 1200$.

This set of preferences seems plausible. But under EU, such preferences cannot hold over a broad range of wealth levels.

Application: Risk Aversion

Loss aversion as an explanation:

Two plausible features of preferences consistent with "loss aversion":

(1) How you feel about absolute gambles is somewhat insensitive to your wealth — e.g., you might reject $(101, .5; -100, .5)$ for all w.

(2) At the same time, scaling outcomes proportionally need not change your preferences much — e.g., you might have

$$\begin{aligned}(12, .5; -10, .5) &\sim (0, 1)\\(120, .5; -100, .5) &\sim (0, 1)\\(1200, .5; -1000, .5) &\sim (0, 1)\end{aligned}$$