ECON 6130: Search and matching

Mathieu Taschereau-Dumouchel

Motivation

We have spent a lot of time on the neoclassical growth model. It is a wonderful model but it has some drawbacks

- That model assumes a unique labor market with market clearing wage
- Does not capture well the process of finding a job
  - Search, vacancy posting, matching, bargaining, etc
- There is no unemployment!
  - Hard to discuss unemployment insurance, scaring effect of unemployment, etc

We turn to a new class of models that takes the details of the labor market seriously

- Peter Diamond, Christopher Pissaride, Dale Mortensen
- Nobel Prize 2010

Overview

References

- LS Chapter 6 and 26
- Rogerson, Shimer and Wright, Search Theoretic Models of the Labor Market: A Survey, Journal of Economic Literature, 2005
- Christopher Pissarides, Equilibrium Unemployment Theory, MIT Press 2000

Math prelim

Nonnegative random variables

- Let p be a random variable with Cumulative Density Function (CDF) $F(P) \equiv \text{Prob}(p \leq P)$.
- Assume $F(0) = 0$ (nonnegativity), $F(\infty) = 1$, F is continuous from the right.
- Assume that there is an upper bound $B < \infty$ such that $F(B) = 1$ (i.e. p is bounded with proba 1).

Recall that the mean of p is given by

$$Ep = \int_0^B p \, dF(p)$$

Let $u = 1 - F(p)$ and $v = p$ we can integrate by parts ($\int_a^b u \, dv = uv|_a^b - \int_a^b v \, du$) so that

$$Ep = \int_0^B p \, dF(p)) = \int_0^B (1 - F(p)) \, dp = B - \int_0^B F(p) dp$$

Math prelim

Consider two *independent* random variables p_1 and p_2 drawn from F and consider the event $\{(p_1 \leq p) \cap (p_2 \leq p)\}$

- $\{(p_1 \leq p) \cap (p_2 \leq p)\}$ happens with probability $(F(p))^2$
- $\{(p_1 \leq p) \cap (p_2 \leq p)\}$ is equivalent to event $\{\max(p_1, p_2) \leq p\}$
- Using our previous result

$$E \max(p_1, p_2) = B - \int_0^B F(p)^2 \, dp$$

- Generalizing with n independent draws from F

$$M_n \equiv E \max(p_1, \ldots, p_n) = B - \int_0^B F(p)^n \, dp$$

Stigler (1961)

Partial equilibrium model of an agent looking for a job

- Risk-neutral agent
- Samples i.i.d. wages from some distribution $F(w)$ with assumptions we made earlier
- Ex-ante decision of how many wages to gather: n
- Getting a wage offer is costly (cost: c)

Stigler (1961)

How many offers to ask for?

The *expected* gain from an additional draw is

$$G_n = M_n - M_{n-1} = B - \int_0^B F(p)^n \, dp - \left(B - \int_0^B F(p)^{n-1} \, dp \right)$$
$$= \int_0^B F(p)^{n-1} \, dp - \int_0^B F(p)^{n-1} F(p) \, dp$$
$$= \int_0^B F(p)^{n-1} (1 - F(p)) \, dp$$

- G_n decreases with n and $\lim_{n \to \infty} G_n = 0$ (right?)
- Optimal rule: pick n such that $G_n \geq c > G_{n+1}$

What's weird with this model?

Stigler (1961)

How many offers to ask for?

The *expected* gain from an additional draw is

$$
\begin{aligned}
G_n = M_n - M_{n-1} &= B - \int_0^B F(p)^n \, dp - \left(B - \int_0^B F(p)^{n-1} \, dp \right) \\
&= \int_0^B F(p)^{n-1} \, dp - \int_0^B F(p)^{n-1} F(p) \, dp \\
&= \int_0^B F(p)^{n-1} (1 - F(p)) \, dp
\end{aligned}
$$

- G_n decreases with n and $\lim_{n\to\infty} G_n = 0$ (right?)
- Optimal rule: pick n such that $G_n \geq c > G_{n+1}$

What's weird with this model? Static search. McCall 1970 fixes this.

Mean-preserving spreads

Consider a class of distributions $F(p, r) = \text{Prob}(P_r \leq p)$ indexed by $r \in \mathbb{R}$

- $F(p, r)$ is differentiable w.r.t. to r for all $p \in ([0, B])$
- There is a finite B such that $F(B, r) = 1$ and $F(0, r) = 0$ for all $r \in R$.

Since $Ep = B - \int_0^B F(p, r)\, dp$, two dist. with same $\int_0^B F(p, r)\, dp$ have same mean.

Mean-preserving spreads

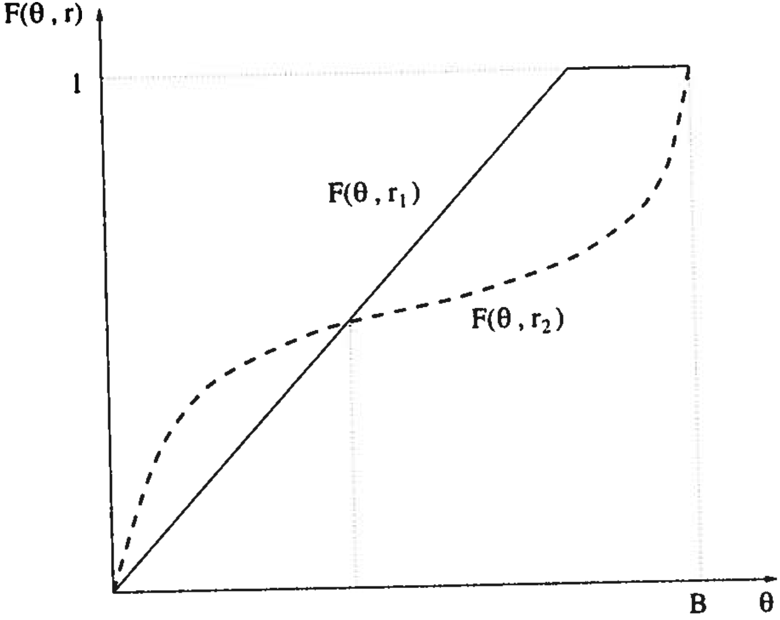
We say that distribution r_2 is a mean-preserving spread of r_1 if

1. Identical means condition

$$\int_0^B (F(\theta, r_1) - F(\theta, r_2))\, d\theta = 0$$

2. Two distributions r_1, r_2 are said to have the single crossing property if there exists a $\hat{\theta}$, with $0 < \hat{\theta} < B$ such that

$$F(\theta, r_2) - F(\theta, r_1) \leq 0\, (\geq 0) \quad \text{when } \theta \geq (\leq) \hat{\theta}$$



F(θ , r)
1
F(θ , r1)
F(θ , r2)
B
θ

Mean-preserving spreads

Properties 1 and 2 imply

$$\int_0^y (F(\theta, r_2) - F(\theta, r_1))\, d\theta \geq 0, \;\; \forall y \in [0, B]$$

For infinitesimal changes in r, an increase in r is said to represent a mean-preserving increase in risk if

$$\int_0^B F_r(\theta, r)\, d\theta = 0$$

and

$$\int_0^y F_r(\theta, r)\, d\theta \geq 0, \;\; 0 \leq y \leq B$$

where $F_r(\theta, r) = \partial F(\theta, r)/\partial r$.

McCall (1970)

An agent searches for a job, taking market conditions as given.

- Each period the agent draws *one* offer w from $F(W) \equiv \text{Prob}\{w \leq W\}$. $F(0) = 0$, $F(B) = 1$ for some $B < \infty$.
- Agent can accept or reject offer. If she rejects, she gets c today and draws another offer tomorrow. If she accepts, she receives w per period forever.

The agent maximizes

$$E \sum_{t=0}^{\infty} \beta^t y_t$$

where $y_t = c$ if she is unemployed and $y_t = w$ if she is employed at wage w.

What is the agent's optimal strategy?

McCall (1970)

Denote by $v(w)$ the expected value of an offer w for an agent who is deciding whether to accept the offer or to reject it. If the agent behaves optimally, we have

$$v(w) = \max\left\{\frac{w}{1-\beta}, c + \beta \int v(w')\, dF(w')\right\}$$

The solution of this Bellman equation is of the form

$$v(w) = \begin{cases} \dfrac{\bar{w}}{1-\beta} = c + \beta \displaystyle\int v(w')\, dF(w') & \text{if } w \leq \bar{w} \\ \dfrac{w}{1-\beta} & \text{if } w \geq \bar{w} \end{cases}$$

where $\bar{w}$ is called the reservation wage. Why?

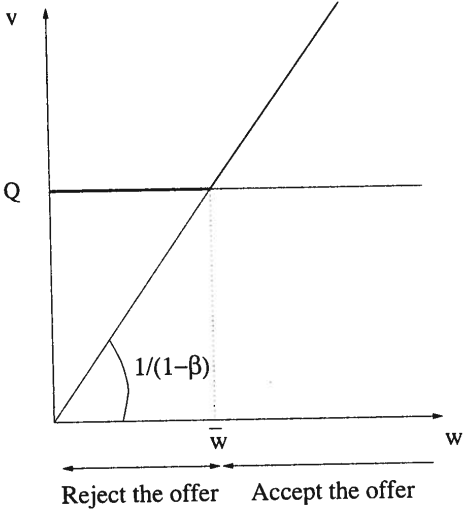


Figure 6.3.1: The function $v(w) = \max\{w/(1-\beta), c + \beta\int_0^B v(w')dF(w')\}$. The reservation wage $\overline{w} = (1-\beta)[c + \beta\int_0^B v(w')dF(w')]$.

McCall (1970)

How do we find $\bar{w}$?

McCall (1970)

How do we find $\bar{w}$? At $w = \bar{w}$ the agent is indifferent

$$\frac{\bar{w}}{1-\beta} = c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta}\, dF(w') + \beta \int_{\bar{w}}^{B} \frac{w'}{1-\beta}\, dF(w')$$

or

$$\frac{\bar{w}}{1-\beta} \int_0^{\bar{w}} dF(w') + \frac{\bar{w}}{1-\beta} \int_{\bar{w}}^{B} dF(w') = c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta}\, dF(w') + \beta \int_{\bar{w}}^{B} \frac{w'}{1-\beta}\, dF(w')$$

or

$$\bar{w} \int_0^{\bar{w}} dF(w') - c = \frac{1}{1-\beta} \int_{\bar{w}}^{B} (\beta w' - \bar{w}) dF(w')$$

Adding $\bar{w} \int_{\bar{w}}^{B} dF(w')$ on both sides:

$$\bar{w} - c = \frac{\beta}{1-\beta} \int_{\bar{w}}^{B} (w' - \bar{w}) dF(w')$$

McCall (1970)

$$
\begin{aligned}
\bar{w} - c =& \frac{\beta}{1-\beta} \int_{\bar{w}}^{B} (w' - \bar{w}) dF(w') \\
=& \beta E \left[\frac{w' - \bar{w}}{1-\beta} | w' \geq \bar{w} \right] Pr(w' \geq \bar{w})
\end{aligned}
$$

- Left-hand side: cost of searching one more time with offer $\bar{w}$ in hand
- Right-hand side: surplus from searching one more time and maybe getting a better offer

These two things must be equal if the agent is optimizing

McCall (1970)

$$\bar{w} - c = \frac{\beta}{1-\beta} \int_{\bar{w}}^{B} (w' - \bar{w}) dF(w')$$

Define the function

$$h(w) \equiv \frac{\beta}{1-\beta} \int_{w}^{B} (w' - w)\, dF(w')$$

Notice that

- $h(0) = Ew\beta/(1-\beta)$
- $h(B) = 0$
- $h(w)$ is differentiable
- $h'(w) = -\frac{\beta}{1-\beta}(1 - F(w)) < 0$
- $h''(w) = \frac{\beta}{1-\beta} F'(w) > 0$

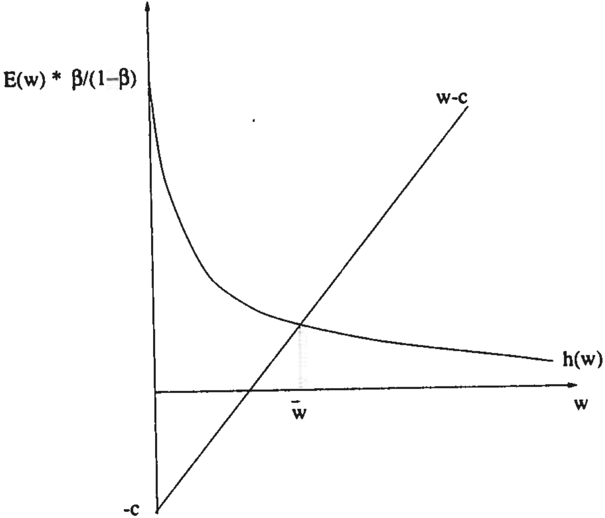


Figure 6.3.2: The reservation wage, $\overline{w}$, that satisfies $\overline{w} - c = [\beta/(1-\beta)] \int_{\overline{w}}^{B} (w' - \overline{w}) dF(w') \equiv h(\overline{w})$.

What happens when the environment changes?

Let's manipulate equations a bit more.

$$
\begin{aligned}
\bar{w} - c &= \frac{\beta}{1-\beta} \int_{\bar{w}}^{B} (w' - \bar{w})\, dF(w') + \frac{\beta}{1-\beta} \int_{0}^{\bar{w}} (w' - \bar{w})\, dF(w') \\
&\quad - \frac{\beta}{1-\beta} \int_{0}^{\bar{w}} (w' - \bar{w})\, dF(w') \\
&= \frac{\beta}{1-\beta} Ew - \frac{\beta}{1-\beta} \bar{w} - \frac{\beta}{1-\beta} \int_{0}^{\bar{w}} (w' - \bar{w})\, dF(w')
\end{aligned}
$$

or

$$
\bar{w} - (1-\beta)c = \beta Ew - \beta \int_{0}^{\bar{w}} (w' - \bar{w})\, dF(w')
$$

Integrating by part

$$
\bar{w} - c = \beta(Ew - c) + \beta \int_{0}^{\bar{w}} F(w')\, dw' \equiv \beta(Ew - c) + \beta g(\bar{w})
$$

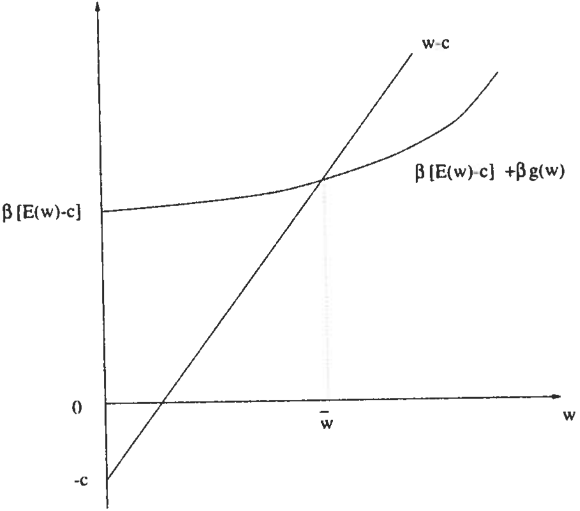


Figure 6.3.3: The reservation wage, $\overline{w}$, that satisfies $\overline{w}-c = \beta(Ew - c) + \beta \int_0^{\overline{w}} F(w')dw' \equiv \beta(Ew - c) + \beta g(\overline{w})$.

McCall (1970)

What happens when:

- c increases? Why?
  - Both curves move to the right $\Rightarrow$ the reservation wage increases
- there is a mean-preserving increase in risk? Why?
  - $g(\bar{w})$ increases $\Rightarrow$ the reservation wage increases
  - Intuition: there is more bad jobs but we don't care about those! The upside is better now so be patient.

McCall (1970)

Problems with this model: What about firm behavior?

1. Workers follow a reservation wage strategy
2. Firms do not gain anything from posting a wage $w > \bar{w}$
3. Firms do not hire anyone if $w < \bar{w}$
4. Therefore $F(w)$ will have a unit mass at $\bar{w}$ (Rothschild 1973)
5. Moreover (Diamond 1971)

$$\begin{aligned}
\bar{w} - c &= \beta(Ew - c) + \beta \int_0^{\bar{w}} F(w')\, dw' \\
\bar{w} - c &= \beta(\bar{w} - c) \\
\bar{w} &= c
\end{aligned}$$

Intuition?

Lucas and Prescott (1974)

We here consider a first model in general equilibrium.

- Continuum of workers populating a large number of separated labor markets (islands)
- Each island has production function $\theta f(n)$ where n is employment ($f' > 0, f'' < 0$, Inada)
- $\theta > 0$ takes m possible values $\theta_1 < \cdots < \theta_m$ with transition probabilities $\pi(\theta, \theta') > 0$

At the beginning of a period, agents are distributed on islands. They observes all productivity and all employments. They decide to move or not.

- If he stays, the worker gets wage $w(\theta, x)$ (x is beginning-of-period labor force)
- If he moves, the worker forgoes labor earning this period and pick today (with full information) another island for next period.

Lucas and Prescott (1974)

Consider first a single island. Its state depends only on θ and x.

- Workers are paid their marginal product $w(\theta, x) = \theta f'[n(\theta, x)]$
- Labor supply $n(\theta, x) \leq x$

Let $v(\theta, x)$ be the value of the optimization problem for an agent finding himself in market (θ, x). Let v_u be the expected value of leaving.

$$v(\theta, x) = \max\{\beta v_u, w(\theta, x) + \beta E[v(\theta', x')|\theta, x]\}$$

Three possibilities

1. $v(\theta, x) = \beta v_u$: some workers leave the market
2. $v(\theta, x) \geq \beta v_u$: no worker is leaving. Some may arrive next period.
3. $v(\theta, x) < \beta v_u$: cannot happen

Lucas and Prescott (1974)

Suppose Case 2 ($n(\theta, x) = x$)

- If no one is leaving but some agents are arriving

$$v(\theta, x) = \theta f'(x) + \beta v_u$$

- If no one is leaving and no one is arriving

$$v(\theta, x) = \theta f'(x) + \beta E[v(\theta', x)|\theta] \leq \theta f'(x) + \beta v_u$$

Putting both cases together

$$v(\theta, x) = \max\left\{\beta v_u, \theta f'(x) + \min\{\beta v_u, \beta E[v(\theta', x)|\theta]\}\right\}$$

Given v_u, this equation is well-behaved with unique solution v. v is nondecreasing in θ and nonincreasing in x.

Lucas and Prescott (1974)

Evolution of island's labor force

1. Some agents leave the market. Then $x' = n(\theta, x)$ solves
$$\theta f'(n(\theta, x)) + \beta E[v(\theta', x')|\theta] = \beta v_u$$
2. No worker is leaving but some will arrive next period. Then x' solves
$$E[v(\theta', x')|\theta] = v_u$$
3. No one is leaving and no one arrives. Then
$$x' = x$$

Combine these rules in a function $\Gamma(\theta', x'|\theta, x)$. Then find stationary distribution from

$$\Psi_{t+1}(\theta', x') = \sum_x \sum_\theta \Gamma(\theta', x'|\theta, x)\Psi_t(\theta, x)$$

We can find v_u from there. (See LS p. 944-946 for details)

GE Models of Job Search

Motivation

- Trade in the labor market is a decentralized economic activity
  - It takes time and effort
  - It is uncoordinated (e.g. no market maker)
- Central points
  - Matching arrangement between employer and employee
  - New matching opportunities constantly arise and disappear

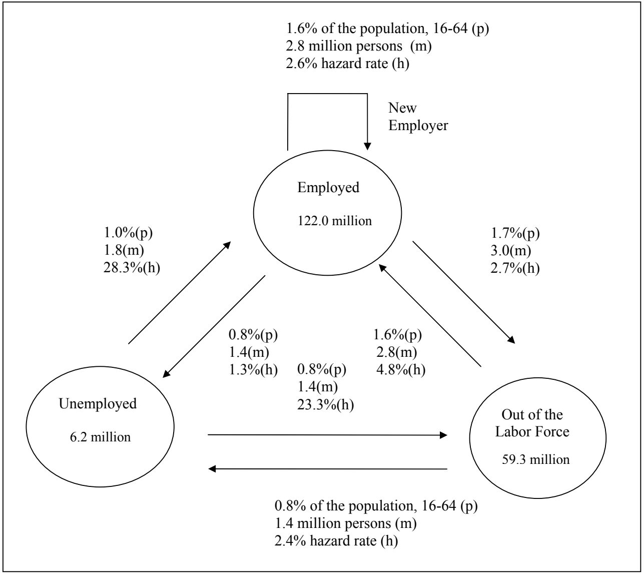
Empirical observations

- Huge amount of labor turnover (see many papers by Davis and Haltiwanger)
- Micro data about the labor market
  - Current population survey (CPS)
  - Job opening and labor turnover survey (JOLTS): 16,000 establishments, monthly
  - Business employment dynamics (BED): entry and exit of establishments.
  - Longitudinal employer household dynamics (LEHD): matched data.

Useful accounting identity (for period t and level of aggregation i):

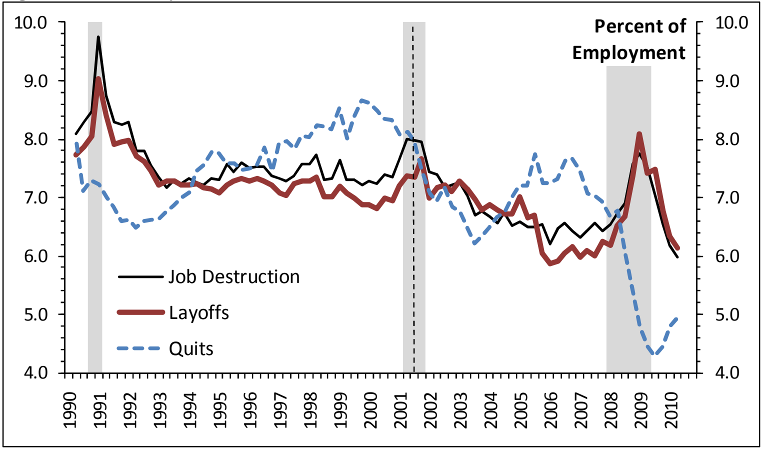
$$\begin{aligned}\text{Net employment change}_{ti} &= \underbrace{\text{Hires}_{ti} - \text{Separations}_{ti}}_{\text{Workers Flow}} \\ &= \underbrace{\text{Creations}_{ti} - \text{Destructions}_{ti}}_{\text{Jobs Flow}}\end{aligned}$$

Figure 1. Average Monthly Worker Flows, Current Population Survey, 1996-2003



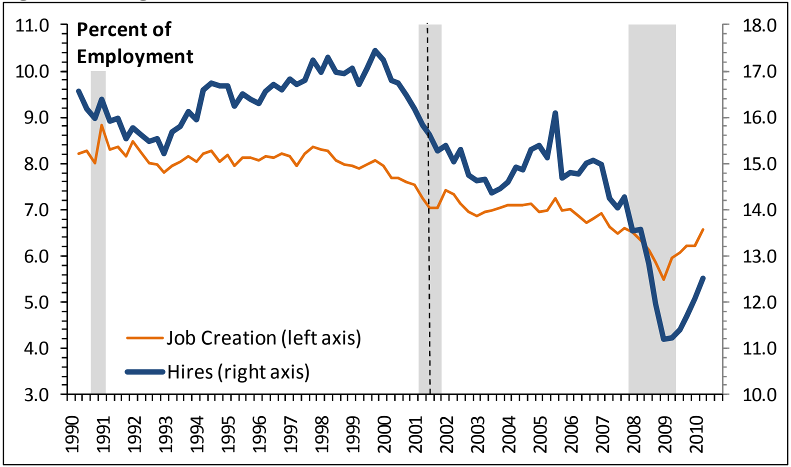
Source: Fallick and Fleischman (2004).

Figure 1. Quits, Layoffs, and Job Destruction



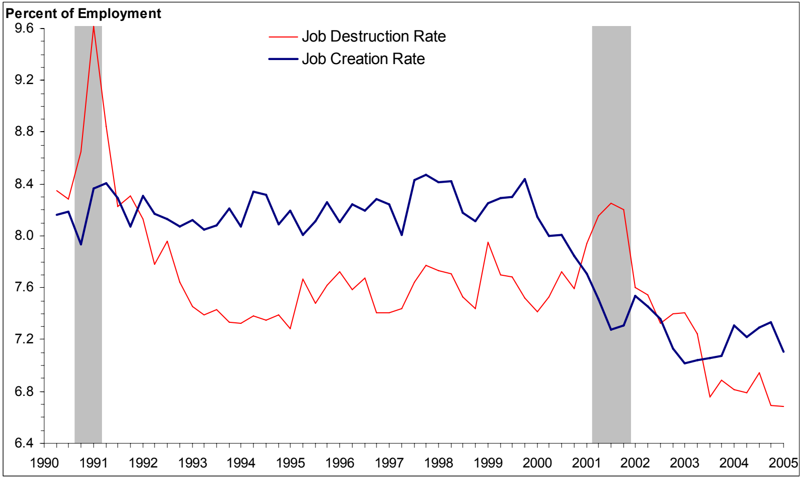
Sources: Quit and layoff rates (2001Q3 – 2010Q2) are authors' calculations using JOLTS establishment microdata weighted to an aggregate value for each quarter using growth rate densities from the BED. Job destruction rates (1990Q2 – 2010Q2) are authors' tabulations directly from the BED data. All estimates are seasonally adjusted. All rates are percentages of employment. Backcasted estimates of the quit and layoff rates are included to the left of the dashed vertical line.

Figure 2. Hiring and Job Creation



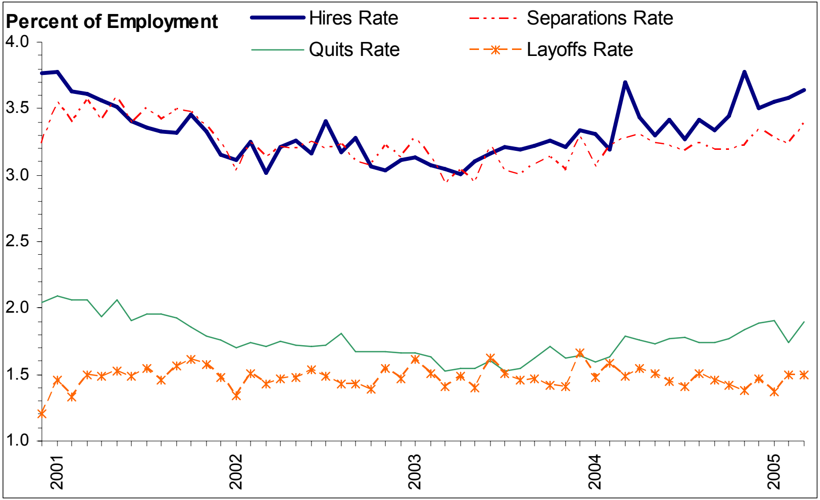
Sources: Hiring rates (2001Q3 – 2010Q2) are authors' calculations using JOLTS establishment microdata weighted to an aggregate value for each quarter using growth rate densities from the BED. Job creation (1990Q2 – 2010Q2) rates are authors' tabulations directly from the BED data. All estimates are seasonally adjusted. All rates are percentages of employment. Backcasted estimates of the hiring rate are included to the left of the dashed vertical line.

Figure 2. Quarterly Job Flows in the Private Sector, 1990-2005



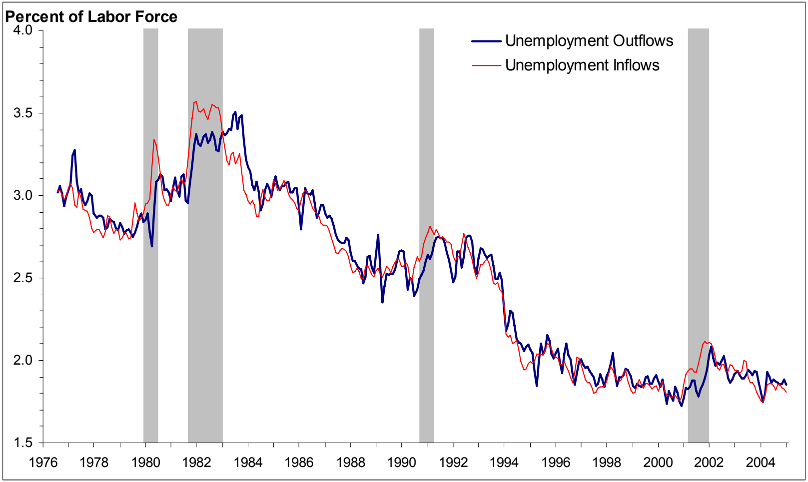
Source: Faberman (2006); tabulated from BLS Business Employment Dynamics (BED) micro data. Shaded areas show NBER-dated recessions.

Figure 4. Monthly Worker Flow Rates, December 2000 to March 2005



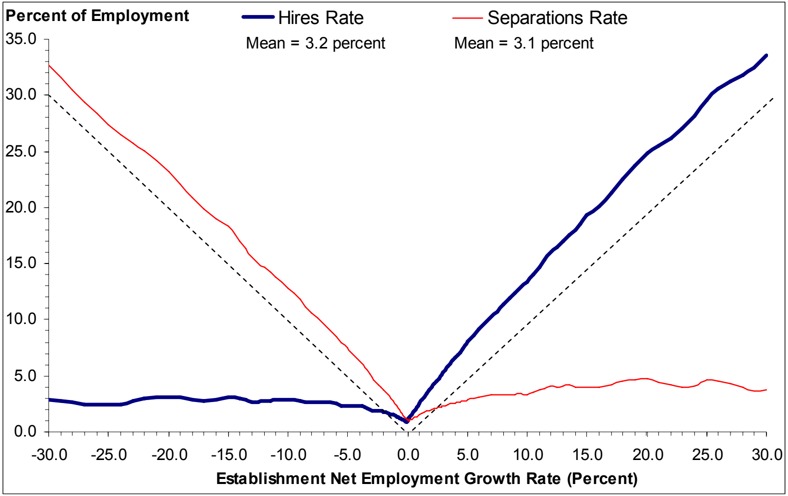
Source: Published data from the BLS Job Openings and Labor Turnover Survey (JOLTS).

Figure 5. Monthly Unemployment Inflows and Outflows, 1976-2005



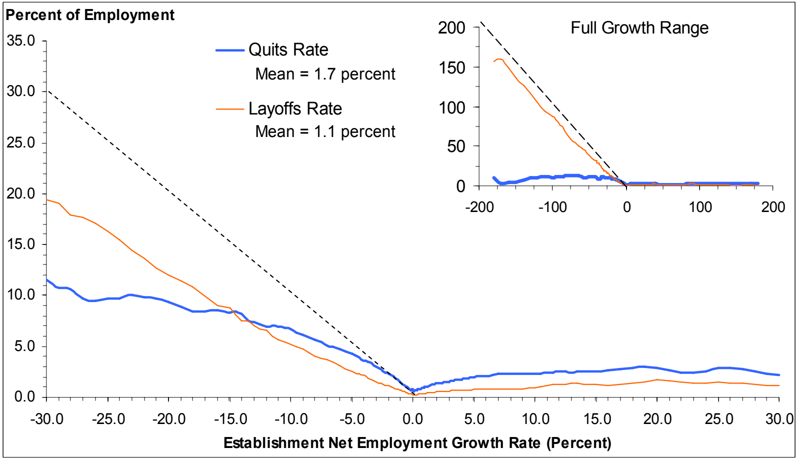
Notes: The figure depicts three-month centered moving averages of estimated gross flows of persons into and out of unemployment based on Current Population Survey (CPS) data. Shaded areas show NBER-dated recessions.

Figure 6: The Relationship of Hires and Separations to Establishment Growth



Notes: The curves are fitted values from nonparametric regressions of establishment-level hires and separations rates (vertical axis) on establishment-level employment growth rates (horizontal axis). The curves are fitted to monthly establishment-level JOLTS data pooled over the period from December 2000 to January 2005.

Figure 7. The Relationship of Quits and Layoffs to Establishment Growth



Notes: The curves are fitted values from nonparametric regressions of establishment-level layoff and quit rates (vertical axis) on establishment-level employment growth rates (horizontal axis). The curves are fitted to monthly establishment-level JOLTS data pooled over the period from December 2000 to January 2005

Table 1. Job and Worker Flow Rates by Sampling Frequency and Data Source

Sampling Frequency and Data Source	Job Creation	Job Destruction	Hires	Separations
Monthly				
JOLTS, continuous monthly units from microdata, Dec-00 to Jan-05	1.5	1.5	3.2	3.1
Quarterly				
JOLTS, continuous quarterly units from microdata, Dec-00 to Jan-05	3.4	3.1	9.5	9.2
BED, all private establishments, 1990:2-2005:1	7.9	7.6	---	---
LEHD, all transitions, ten selected states, 1993:2-2003:3	7.0	6.0	25.0	24.0
LEHD, "full-quarter" transitions, ten selected states, 1993:2-2003:3	7.6	5.2	13.1	10.7
Annual				
BED, from Pinkston and Spletzer (2004), private establishments, 1998-2002	14.6	13.7	---	---

Table 2. Job and Worker Flows by Selected Industries

A. Average Quarterly Job Flow Rates in the BED, 1990:2 – 2005:1

	Job Creation	*Job Destruction*	*Net Growth*
Total Private	7.9	7.6	0.3
Construction	14.3	13.9	0.4
Manufacturing	4.9	5.3	-0.4
Retail Trade	8.1	7.9	0.2
Professional & Business Services	9.9	9.1	0.8
Leisure & Hospitality	10.7	10.2	0.5

B. Average Monthly Worker Flow Rates in JOLTS, December 2000 to January 2005

					Layoffs Per	
	Hires	*Separations*	*Quits*	*Layoffs*	*Quit*	*Destroyed Job*
Total Nonfarm	3.2	3.1	1.7	1.1	0.7	0.8
Construction	5.3	5.5	2.1	3.2	1.5	1.1
Manufacturing	2.2	2.7	1.2	1.2	1.1	0.8
Retail Trade	4.3	4.2	2.6	1.3	0.5	0.7
Professional & Business Services	4.2	3.9	2.0	1.6	0.8	1.0
Leisure & Hospitality	6.1	5.9	3.9	1.8	0.5	0.7

Notes: Estimates based on authors' tabulations of BED and JOLTS microdata. Rates are percentages of employment, calculated as described in the text.

Some distributions

A **Poisson distribution** is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed interval of time if these events occur with known average rate and *independently* of the time since last event.

If the expected number of occurrences in a given interval is λ, then the probability of $k \in \mathbb{N}$ occurrences is equal to

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}.$$

Example: if the events occur on average 4 times per minute, and one is interested in the probability of an event occurring k times in a 10 minute interval, one would use a Poisson distribution with $\lambda = 10 \times 4 = 40$.

An **exponential distribution** with mean λ^{-1} has pdf

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Poisson process

A **Poisson process** with parameter λ is a continuous-time stochastic counting process $\{N(t), t \geq 0\}$ such that:

1. $N(0) = 0$
2. The number of occurrences in disjoint time intervals is independent
3. The probability distribution of the number of occurrences in any time interval only depends on the length of the interval
4. No occurrences are simultaneous.

Then:

1. The probability distribution of $N(t)$ is a *Poisson distribution*. More generally

$$P[N(t+\tau) - N(t) = k] = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$$

2. The probability distribution of the waiting time until the next occurrence is an *exponential distribution* with mean λ^{-1}.

Random matching

We will look at a few models of random matching. First, Pissarides (1985).

Environment

- Continuous time
- Constant and exogenous interest rate r

Workers

- Continuum of measure L of identical agents
- Linear preferences with discounted utility

$$\int_0^\infty e^{-rt} y(t)\, dt$$

  where $y(t)$ is income per unit of time at time t

Firms

- Endogenous number of firms (one firm $=$ one job) (but linearity...)
- Competitive producers of the final output
- Free entry. Firms enter until expected profit from entering is 0
- Vacancy cost $c > 0$ per unit of time

Labor market

Matches occur randomly between unemployed agents and vacancies

- L agents (employed and unemployed), U unemployed agents and V vacancies. Total number of matches is given by the matching function

$$m(U, V) = m(uL, vL) \equiv fL$$

where u unemployment rate, v is the vacancy rate and f is the rate of jobs (matches) creations.

- Assume that m is increasing, concave and constant returns to scale:

$$f = m(u, v)$$

- Microfoundations? Butters 1977, Shimer 2007.
- Externalities?

Labor market

- It is convenient to define a labor market tightness

$$\theta \equiv \frac{v}{u}$$

- Then,

$$q(\theta) \equiv \frac{m(U, V)}{V} = m\left(\frac{u}{v}, 1\right) = m\left(\frac{1}{\theta}, 1\right)$$

We can show that

- $q'(\theta) \leq 0$

Labor market

Since $\frac{f}{v} = \frac{m(u,v)}{v} = q(\theta)$, we have that

- $q(\theta)$ is the Poisson rate at which vacancies are filled
- $1/q(\theta)$ is the mean duration of a vacancy

Since $\frac{f}{u} = \frac{m(u,v)}{u} = \theta q(\theta)$, we have that

- $\theta q(\theta)$ is the Poisson rate at which unemployed agents find a job
- $1/[\theta q(\theta)]$ is the mean duration of unemployment

Note that all the rates depend uniquely on θ and that prices do not affect these rates directly.

Job creation and Job destruction

A job is created when a worker and firm meet and agree on a wage

- In one period, $fL = \theta q(\theta) uL$ jobs are created
- Job creation rate is $\frac{u\theta q(\theta)}{1-u}$
- Job destruction is assumed exogenous at Poisson rate δ
- In one period, $\delta(1-u)L$ jobs are destroyed
- Job destruction rate: $\frac{\delta(1-u)}{1-u} = \delta$

Dynamics of unemployment

Dynamics of unemployment

$$\frac{du}{dt} \equiv \dot{u} = \delta(1-u) - u\theta q(\theta)$$

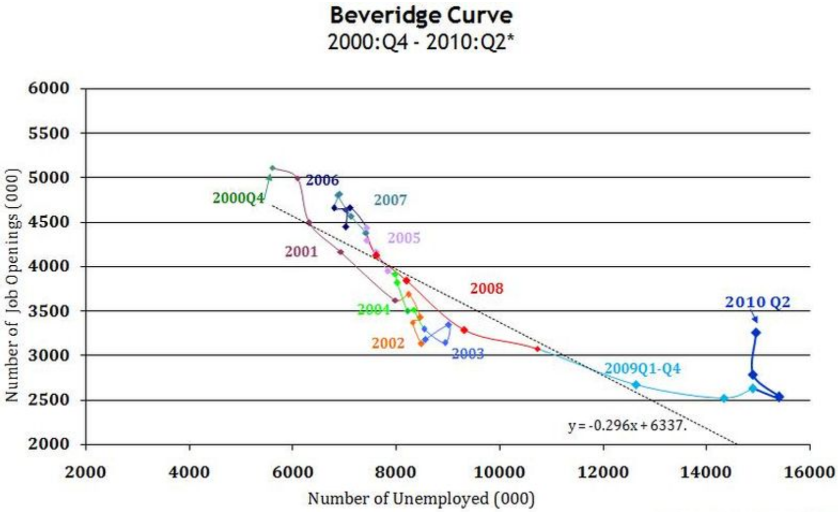
In steady state ($\dot{u} = 0$:)

$$\delta(1-u) = u\theta q(\theta)$$

or

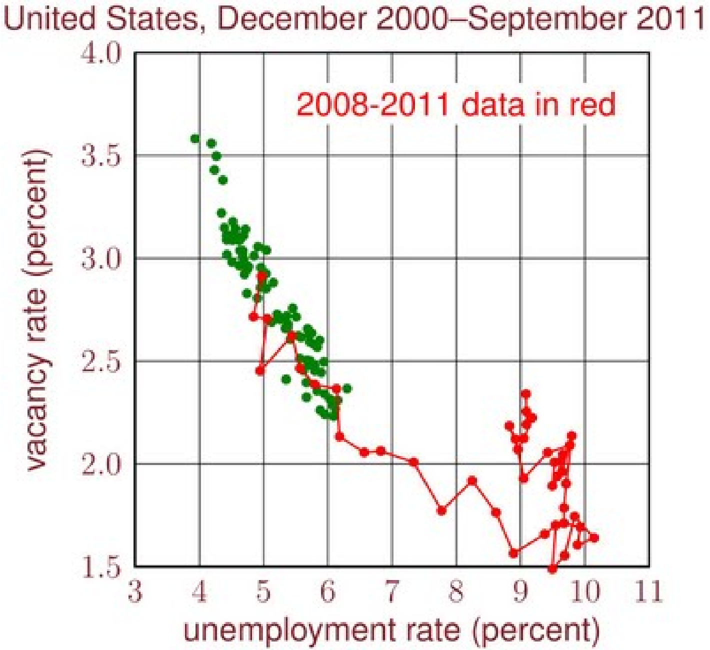
$$\boxed{u = \frac{\delta}{\delta + \theta q(\theta)}}$$

This last equation describes the Beveridge curve.



Source: BLS, Job Openings and Labor Turnover Survey and Current Population Survey

* Q2:2010 is average of Apr & Ma


United States, December 2000–September 2011
2008-2011 data in red
vacancy rate (percent)
unemployment rate (percent)
4.0
3.5
3.0
2.5
2.0
1.5
3
4
5
6
7
8
9
10
11

Wages and the value of the firm

- Wage w (to be determined in a second)
- Hours fixed and normalized to 1. Productivity p.
- Either party can break the contract at any time
- J is the value function of an occupied job and V is the value function of a vacant job

Then in a stationary equilibrium

$$rV = -c + q(\theta)(J - V)$$
$$rJ = p - w + \delta(V - J)$$

Where do these equations come from? Suppose discrete time with each period lasting $\Delta > 0$

$$V = -c\Delta + (1 - e^{-q(\theta)\Delta} q(\theta)\Delta) e^{-r\Delta} V + e^{-q(\theta)\Delta} q(\theta)\Delta e^{-r\Delta} J$$
$$V\left(\frac{1 - e^{-r\Delta}}{\Delta}\right) = -c + e^{-q(\theta)\Delta} q(\theta) e^{-r\Delta}(J - V)$$

Taking the limit $\Delta \to 0$ we get our equation. (non stationary case?)

Wages and the value of the firm

Because of free entry (in equilibrium), we have

$$V = 0$$

$$J = \frac{p - w}{r + \delta}$$

$$J = \frac{c}{q(\theta)}$$

$$\frac{dJ(w)}{dw} \equiv J' = -\frac{1}{r + \delta}$$

Then

$$\boxed{\frac{p - w}{r + \delta} = \frac{c}{q(\theta)}}$$

This equation is the job creation condition. Intuition?

Workers

- Value of not working z (leisure, home production, unemployment insurance)
- U is the value function of unemployed workers
- V is the value function of employed workers

Then in a stationary equilibria

$$rU = z + \theta q(\theta)(W - U)$$

$$rW = w + \delta(U - W)$$

Note that $W' = \frac{1}{r+\delta}$. Also

$$W - U = \frac{w - rU}{r + \delta}$$

Nash bargaining

From Nash (1950): two players split a pie of size x. We want a system that is

1. Invariant to equivalent utility representations
2. Pareto optimal
3. Independent to irrelevant alternatives
4. Symmetry

Then:

$$w = \arg\max_{w} [u(w) - u(d)][v(x - w) - v(d)]$$

where d represents the status quo. Also: extensive game formulation. In our case, the wage will be decided by generalized Nash bargaining:

$$w = \arg\max (W - U)^{\beta} (J - V)^{1-\beta}$$

Nash bargaining

Taking the FOC

$$\beta\frac{W'}{W-U} = -(1-\beta)\frac{J'}{J-V}$$

and then:

$$W = U + \beta\underbrace{(W-U+J)}_{\text{Total match surplus}} = U + \beta S$$

Also

$$W - U = \frac{\beta}{1-\beta}J$$

Since $J = \frac{p-w}{r+\delta}$ and $W - U = \frac{w-rU}{r+\delta}$

$$\frac{w-rU}{r+\delta} = \beta\left(\frac{w-rU}{r+\delta} + \frac{p-w}{r+\delta}\right)$$

and

$$w = rU + \beta(p - rU)$$

Nash bargaining

Note that

$$w = (1-\beta)rU + \beta p$$

$$w = (1-\beta)(z + \theta q(\theta)(W - U)) + \beta p$$

$$w = (1-\beta)\left(z + \theta q(\theta)\frac{\beta}{1-\beta}\frac{c}{q(\theta)}\right) + \beta p$$

$$\boxed{w = (1-\beta)z + \beta(p + \theta c)}$$

Full model

Three equations characterize the full model

$$w = (1-\beta)z + \beta(p + \theta c)$$

$$\frac{p-w}{r+\delta} = \frac{c}{q(\theta)}$$

$$u = \frac{\delta}{\delta + \theta q(\theta)}$$

To find the full equilibrium, combine the first two equations

$$(1-\beta)(p-z) = \frac{r + \delta + \beta\theta q(\theta)}{q(\theta)} c$$

and plot in the Beveridge diagram.

What if we change z, β, p?

Efficiency

Is the allocation efficient? First, optimal control review.

$$\max_{u \in U} \int_0^{\infty} L(x(t), u(t))\, dt$$

subject to

$$\dot{x} = f(x, u)$$

$$x(0) = x_0$$

Construct the **Hamiltonian**

$$H(\lambda(t), x(t), u(t), t) = L(x(t), u(t)) - \lambda(t) f(x(t), u(t))$$

Pontryagin's maximum principle:

$$\frac{\partial H}{\partial x} = \dot{\lambda}(t) \text{ and } \frac{\partial H}{\partial u} = 0$$

http://elsa.berkeley.edu/~obstfeld/ftp/perplexed/cts4a.pdf

Efficiency

The social planner faces the same friction as the agents. Its problem is:

$$\max_{u,\theta} \int_0^\infty e^{-rt}(p(1-u) + zu - c\theta u)$$

$$s.t.\ \dot{u} = \delta(1-u) - u\theta q(\theta)$$

Using Pontryagin's maximum principle:

$$e^{-rt}(p - z + c\theta) - \mu(\delta + \theta q(\theta)) = -\dot{\mu}$$

$$e^{-rt}cu = \mu u q(\theta)(1 - \eta(\theta))$$

where μ is the costate (multiplier) and $\eta(\theta)$ is the elasticity of $q(\theta)$

$$\eta(\theta) \equiv -\frac{\theta q'(\theta)}{q(\theta)}$$

Efficiency

Assume that all real variables are in steady state. Then, from the second equation, we get

$$\dot{\mu} = -r\mu$$

and (after manipulations)

$$(1 - \eta(\theta))(p - z) - \frac{r + \delta + \eta(\theta)\theta q(\theta)}{q(\theta)} c = 0$$

But, in the competitive economy we found

$$(1 - \beta)(p - z) = \frac{r + \delta + \beta\theta q(\theta)}{q(\theta)} c$$

Efficiency

Assume that all real variables are in steady state. Then, from the second equation, we get

$$\dot{\mu} = -r\mu$$

and (after manipulations)

$$(1 - \eta(\theta))(p - z) - \frac{r + \delta + \eta(\theta)\theta q(\theta)}{q(\theta)} c = 0$$

But, in the competitive economy we found

$$(1 - \beta)(p - z) = \frac{r + \delta + \beta\theta q(\theta)}{q(\theta)} c$$

Therefore, the economy is efficient if and only if

$$\boxed{\eta(\theta) = \beta}$$

Efficiency

Many authors use a Cobb-Douglas structure for the matching function

$$m = Au^{\eta} v^{1-\eta}$$

Then: $\eta(\theta) = \eta$ and efficiency requires $\eta = \beta$. (Hosios, 1990)

- If $\eta > \beta$ equilibrium unemployment is below its social optimum
- If $\eta < \beta$ equilibrium unemployment is above its social optimum

Intuition: externalities equal share of surplus.

Mortensen and Pissarides (1994)

Similar to the previous model but with endogenous job destruction

- Productivity of a job px where x is the idiosyncratic part
- New x arrives with Poisson rate λ
- Distribution of x is $G(x)$
- Distribution is memoriless (Poisson process) and with bounded support $[0, 1]$
- Initial draw is $x = 1$. Why?

The discounted expected value of a job is now $J(x)$. Then

1. if $J(x) \geq 0$ the job is kept
2. if $J(x) < 0$ the job is destroyed

There exist an R (reservation productivity) such that $J(R) = 0$. Why?

Unemployment flows

- A law of large number applies to the aggregate economy
- Job destruction: $\lambda G(R)(1-u)$
- Unemployment evolves

$$\dot{u} = \lambda G(R)(1-u) - u\theta q(\theta)$$

- In steady state:

$$\boxed{u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}}$$

Value functions

Value functions for the firm

$$rV = -c + q(\theta)(J(1) - V)$$

$$rJ(x) = px - w(x) + \lambda \left(\int_R^1 J(s)dG(s) + G(R)V - J(x) \right)$$

Value functions for the workers

$$rU = z + \theta q(\theta)(W(1) - U)$$

$$rW(x) = w(x) + \lambda \left(\int_R^1 W(s)dG(s) + G(R)U - W(x) \right)$$

By free entry $V = 0$ and $J(1) = \frac{c}{q(\theta)}$.

Wages are still set by Nash-bargaining

$$W(x) - U = \beta(W(x) - U + J(x))$$

Solving the model

Repeating the same steps as in Pissarides, we find the equilibrium wage

$$\boxed{w(x) = (1-\beta)z + \beta(px + \theta c)}$$

To find the reservation productivity

$$W(R) = U \text{ and } J(R) = 0$$

From the firm's value function

$$rJ(x) = px - (1-\beta)z - \beta(px + \theta c) + \lambda \int_R^1 J(s)dG(s) - \lambda J(x)$$

and then

$$(r+\lambda)J(x) = (1-\beta)px - (1-\beta)z - \beta\theta c + \lambda \int_R^1 J(s)dG(s)$$

at $x = R$

$$(r+\lambda)J(R) = (1-\beta)pR - (1-\beta)z - \beta\theta c + \lambda \int_R^1 J(s)dG(s) = 0$$

Solving the model

Thus we have

$$
\begin{aligned}
(r+\lambda)J(x) &= (1-\beta)p(x-R) \\
(r+\lambda)J(1) &= (1-\beta)p(1-R) \\
(r+\lambda)\frac{c}{q(\theta)} &= (1-\beta)p(1-R)
\end{aligned}
$$

So that, in equilibrium, we find the **job creation condition**

$$
\boxed{(1-\beta)p\frac{1-R}{r+\lambda} = \frac{c}{q(\theta)}}
$$

Solving the model

Note that:

$$(r+\lambda)J(x) = (1-\beta)p(x-R) \Rightarrow J(x) = \frac{1-\beta}{r+\lambda}p(x-R)$$

Then, since

$$(r+\lambda)J(x) = (1-\beta)(px-z) - \beta\theta c + \lambda\int_R^1 J(s)dG(s)$$

we find

$$(r+\lambda)J(x) = (1-\beta)(px-z) - \beta\theta c + \frac{\lambda(1-\beta)p}{r+\lambda}\int_R^1 (s-R)dG(s)$$

and since $J(R) = 0$, we find the **job destruction condition**

$$\boxed{R - \frac{z}{p} - \frac{\beta}{1-\beta}\theta c + \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s) = 0}$$

The boxed equations fully characterize the steady-state equilibrium.

Efficiency

Social welfare in this model

$$\max_{u,\theta} \int_0^\infty e^{-rt}(y + zu - c\theta u)dt$$

subject to

$$\dot{u} = \lambda G(R)(1-u) - u\theta q(\theta)$$

and where y, the average product per person in the labor market follows

$$\dot{y} = p\theta q(\theta)u + \lambda(1-u)\int_R^1 psdG(s) - \lambda y$$

Surprisingly (or not?) Hosios' condition still applies. (see Pissarides' book for details)

Mortensen and Pissarides (1994)

Problems with Mortensen and Pissarides (1994)?

- Wage dispersion: different wages for the same work (observable and unobservable heterogeneity)
- Evidence of wage dispersion: Mincerian wage regression

$$w_i = X_i' \beta + \epsilon_i$$

- Typical Mincerian regression accounts for 25-30% of variation in the data.

We want a model that will explain the dispersion. Theoretical challenge:

- Diamond's paradox shows up easily
- Wage dispersion you get from Mortensen-Pissarides is very small (Krusell, Hornstein, Violante, 2007).

Burdett and Mortensen (1998) uses on-the-job search to generate dispersion

Burdett and Mortensen (1998)

Basic idea:

- If workers also search while being employed and not only while being unemployed, firms that pay a higher wages can hire not only unemployed workers but also employed workers that earn less
- The number of employed workers will therefore increase with the wage offered, such that in equilibrium high wage firms with low profit per worker but large number of employees make the same profit as low wage firms that earn a high profit per worker but have only few workers.

Burdett and Mortensen (1998)

Environment

- Unit measure of identical workers
- Measure of identical firms (firms can have more than 1 worker)
- Each worker is unemployed (state 0) or employed (state 1)
- Endogenous Poisson arrival rate of new offers λ. Same for workers and unemployed agents.
- All offers come from a distribution F to be determined in equilibrium

As before

- Matches are destroyed at exogenous rate δ
- Values of not working z
- Discount rate r
- Vacancy cost c

Value function for workers

Utility of unemployed agent

$$rV_0 = z + \lambda \left[\int \max\{V_0, V_1(w')\} dF(w') - V_0 \right]$$

Utility of worker employed at wage w

$$rV_1(w) = w + \lambda \int \left[\max\{V_1(w), V_1(w')\} - V_1(w) \right] dF(w') + \delta [V_0 - V_1(w)]$$

For unemployed workers, there is a reservation wage w_R such that $V_0 = V_1(w_R)$. (why?)

Clearly, $w_R = z$. (why?)

For employed workers, the reservation wage is equal to the current wage.

Firms problem

$G(w)$ is the distribution of wages of employed workers

Firm posts a wage (Butters 1977, Burdett and Judd 1983, and Mortensen 1990). The value of posting a wage w:

$$\pi(p, w) = \frac{u + (1-u)G(w)}{r + \delta + \lambda(1 - F(w))}(p - w)$$

- Firms sets wage to maximize $\pi(p, w)$. No symmetric pure strategy equilibrium. (why?)
- Firms will never post w lower than z. (why?)

Unemployment

Steady state unemployment

$$\lambda(1 - F(z))u = \delta(1 - u)$$

then

$$u = \frac{\delta}{\delta + \lambda[1 - F(z)]} = \frac{\delta}{\delta + \lambda}$$

where we have used the fact that no firm will post wage lower than z

Distribution of workers

Agents gaining less than w

$$E(w) \equiv (1-u)G(w)$$

Then

$$\dot{E}(w) = \lambda F(w)u - (\delta + \lambda[1-F(w)])E(w)$$

In steady state:

$$E(w) = \frac{\lambda F(w)}{\delta + \lambda[1-F(w)]}u$$

which yields

$$G(w) = \frac{E(w)}{1-u} = \frac{\delta F(w)}{\delta + \lambda[1-F(w)]}$$

Solving for an equilibrium

Equilibrium objects: $\lambda, u, F(w), G(w)$

- $F(w)$ does not have mass points and has a connected support
- By free entry

$$\pi(p,z) = \boxed{\frac{\delta}{\delta+\lambda}\frac{p-z}{r+\delta+\lambda} = c}$$

  which we can solve for λ
- Once we have λ, we have $u = \frac{\delta}{\delta+\lambda}$

Using the equality of profit and some manipulation:

$$\pi(p,w) = \frac{\left[\frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda}\frac{\delta F(w)}{\delta+\lambda[1-F(w)]}\right]}{r+\delta+\lambda(1-F(w))}(p-w)$$

$$\frac{\delta}{\delta+\lambda[1-F(w)]}\frac{p-w}{r+\delta+\lambda(1-F(w))} = \frac{\delta}{\delta+\lambda}\frac{p-z}{r+\delta+\lambda}$$

Solving for an equilibrium

The previous equation is quadratic in $F(w)$. Set $r = 0$ for simplicity:

$$F(w) = \frac{\delta + \lambda}{\delta} \left[1 - \left(\frac{p - w}{p - z} \right)^{1/2} \right]$$

and

$$G(w) = \frac{\delta}{\lambda} \left[\left(\frac{p - w}{p - z} \right)^{1/2} - 1 \right]$$

Highest wage is $F(w^{\max}) = 1$

$$w^{\max} = \left(1 - \frac{\delta}{\delta + \lambda} \right)^2 p + \left(\frac{\delta}{\delta + \lambda} \right)^2 z$$

Competitive search - Moen (1997)

Competitive search - Moen (1997)

- A market maker chooses a number of markets m (indexed by i) and determines the wage w_i in each submarket.
- Workers and firms are free to move between markets.

Workers

Value functions

$$rU_i = z + \theta_i q(\theta_i)(W_i - U_i)$$
$$rW_i = w_i + \delta(U_i - W_i)$$

Reorganizing

$$W_i = \frac{1}{r+\delta} w_i + \frac{\delta}{r+\delta} U_i$$
$$rU_i = z + \theta_i q(\theta_i) \left(\frac{w_i - rU_i}{r+\delta} \right)$$

- Workers pick the market with the highest U_i
- In equilibrium, all <u>open</u> submarkets deliver the same U_i:

$$\theta_i q(\theta_i) = \frac{rU - z}{w_i - rU}(r+\delta)$$

- Negative relation between wage and labor market tightness.
- If $w_i < rU$ the market will not attract workers and will close.

Firms

Value functions

$$
\begin{aligned}
rV_i &= -c + q(\theta_i)(J_i - V_i) \\
rJ_i &= p - w_i - \delta J_i
\end{aligned}
$$

Thus

$$
rV_i = -c + q(\theta_i)\left(\frac{p - w_i}{r + \delta} - V_i\right)
$$

Each firm solves

$$
rV_i = \max_{w_i,\theta_i}\left(-c + q(\theta_i)\left(\frac{p - w_i}{r + \delta} - V_i\right)\right)
$$

$$
\text{s.t. } rU_i = z + \theta_i q(\theta_i)\left(\frac{w_i - rU}{r + \delta}\right)
$$

Equilibrium

Impose free-entry condition $V_i = 0$ and solving the dual:

$$rU_i = \max_{w_i, \theta_i} \left(z + \theta_i q(\theta_i) \frac{w_i - rU}{r + \delta} \right)$$

$$\text{s.t. } c = q(\theta_i) \frac{p - w_i}{r + \delta}$$

Plugging the value of w_i from the constraint into the objective function

$$rU_i = \max_{\theta_i} \left(z - c\theta_i + \theta_i q(\theta_i) \frac{p - rU}{r + \delta} \right)$$

Solution

$$c = q(\theta_i) \frac{p - rU}{r + \delta} + \theta_i q'(\theta_i) \frac{p - rU}{r + \delta}$$

that is unique if $\theta_i q(\theta_i)$ is concave.

This equilibrium is efficient. We can extend this model in many directions.

Search-theoretic model of money

Motivation

- What is money?
- Why do we use money as a society
- Use search theory to model existence of money
- Contrast with other approaches:
  1. Money in DSGE models (cash in advance, money in utility function)
  2. Money in OLG models

Search-theoretic model of money

Three reasons for money:

1. Double-coincidence of wants problem.
2. Long-run commitment cannot be enforced.
3. Agents are anonymous: histories are not public information.

Three generations of models

1. 1 unit of money, 1 unit of good: Kiyotaki and Wright (1993).
2. 1 unit of money, endogenous units of good: Trejos and Wright (1995).
3. Endogenous units of money, endogenous units of good: Lagos and Wright (2005).

We will look at the first one.

Environment

- $[0,1]$ continuum of anonymous agents
- Live forever and discount future at rate r
- $[0,1]$ continuum of goods. Good i is produced by agent i
- Goods are non-storable
- Unit cost of production $c \geq 0$

Environment

Double-coincidence of wants problem

- I do not produce what I like
- iWj: agent i likes to consume good produced by agent j:
  1. utility $u > c$ from consuming j
  2. utility of 0 otherwise
- Probabilities of matching

$$p(iWi) = 0$$

$$p(jWi) = x$$

$$p(jWi|iWj) = y$$

Environment

Fixed money and fixed good

- Exogenous quantity $M \in [0, 1]$ of an indivisible unit of storable good
- Holding money yields zero utility
- Initial endowment: M agents are randomly endowed with one unit of money
- Agents holding money cannot produce

Trades

- Pairwise random matching of agents with Poisson rate α
- Upon meeting, agents decide to trade or not. Then, they part company and re-enter process.
- History of previous trades is unknown
- Exchange 1 unit of good for 1 unit of good (barter) or 1 unit of money

Individual trading strategies

Individual trading strategies

- Agents never accept a good in trade if he does not like to consume it since it is not storable.
- They will barter if they like each other goods.
- Would they accept money for goods and vice versa?
- We will look at stationary and symmetric Nash equilibria.

Suppose you have a unit of money:

- You meet someone with arrival rate α
- This person can produce with probability $1 - M$
- With probability x you like what he produces.
- With probability $\pi = \pi_0 \pi_1$ (endogenous objects to be determined) both of you want to trade.
- If $\pi > 0$, we say that money circulates.

Value functions

Value function with money, V_1:

$$rV_1 = \alpha x(1 - M)\pi(u + V_0 - V_1)$$

Value function without money, V_0:

$$rV_0 = \alpha xy(1 - M)(u - c) + \alpha xM\pi(V_1 - V_0 - c)$$

Renormalize $\alpha x = 1$ by picking time units:

$$rV_1 = (1 - M)\pi(u + V_0 - V_1)$$

$$rV_0 = y(1 - M)(u - c) + M\pi(V_1 - V_0 - c)$$

Individual trading strategy:

- Net gain from trading goods for money

$$\Delta_0 = V_1 - V_0 - c = \frac{(1 - M)(\pi - y)(u - c) - rc}{r + \pi}$$

- Net gain from trading money from goods:

$$\Delta_1 = u + V_0 - V_1 = \frac{(M\pi + (1 - M)y)(u - c) + ru}{r + \pi}$$

Equilibrium

Equilibrium Conditions for π_0 and π_1:

$$\pi_j = 1 \text{ if } \Delta_j > 0$$
$$\pi_j \in [0,1] \text{ if } \Delta_j = 0$$
$$\pi_j = 0 \text{ if } \Delta_j < 0$$

- Clearly $\Delta_1 > 0$. Hence $\pi_1 = 1$, the agent with money always wants to trade.
- For π_0, we have

$$\Delta_0 = \frac{(1-M)(u-c)\pi_0}{r+\pi_0} - \frac{(1-M)y(u-c)+rc}{r+\pi_0}$$

- Then Δ_0 has the same sign as

$$\pi_0 - \frac{rc+(1-M)y(u-c)}{(1-M)(u-c)} = \pi_0 - \hat{\pi}$$

Multiple Equilibria

- Nonmonetary equilibrium: we have an equilibrium where $\pi_0 = 0$
- Monetrary equilibrium: if

  $$c < \frac{(1-M)(1-y)}{r + (1-M)(1-y)}$$

  then $\hat{\pi} < 1$ and $\pi_0 = 1$ is an equilibrium as well.
- Mixed-monetary equilibrium $\pi_0 = \hat{\pi}$

Welfare

- Define welfare as the average utility:

$$W = MV_1 + (1 - M)V_0$$

- Then:

$$rW = (1 - M)[(1 - M)y + m\pi](u - c)$$

- Note that welfare is increasing in π

Welfare

Welfare with $\pi = 1$

- Note:

$$rW = (1 - M)[(1 - M)y + M](u - c)$$

- Maximize W with respect to M:

$$M^* = \frac{1 - 2y}{2 - 2y} \text{ if } y < 1/2$$
$$M^* = 0 \text{ if } y \geq 1/2$$

- Intuition: facilitate trade versus crowding out barter

Welfare with $\pi = 0$

- Note:

$$rW = (1 - M)[(1 - M)y](u - c)$$

- Monotonically decreasing in M, optimum: $M^* = 0$
- Intuition?