ECON 6130: Problem set 2

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Please upload your copy on Canvas. You may work in groups, but you must turn in your own answers. Actively working on the assignments is *absolutely essential* for your understanding of the course material.

Problem 1. *Consider a pure exchange economy in which there 3 agents who live forever. Time is discrete an indexed by* $t = 0, 1, 2, \dots$. *In each period, the agents trade a nonstorable consumption good. Agents have symmetrical preferences represented by the utility function*

$$u(c^i) = \sum_{t=0}^{\infty} \beta^t \log(c_t^i)$$

with $\beta \in (0,1)$ *and where* c^i *denotes the consumption steam of agent* $i \in \{1,2,3\}$. *Agent* i *receives the deterministic endowment*

$$e_t^1 = \begin{cases} 3 & \text{if } t = \{0, 3, 6, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

$$e_t^2 = \begin{cases} 3 & \text{if } t = \{1, 4, 7, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

$$e_t^3 = \begin{cases} 3 & \text{if } t = \{2, 5, 8, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

1. What is the aggregate endowment in each period?

2. Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with who, and so on. Define a competitive Arrow-Debreu equilibrium for this economy. Use the notation $\hat{p}_t$ for the equilibrium prices and $\hat{c}_t^i$ for the equilibrium consumptions.

3. Describe a sequential markets structure for this economy, explaining when markets are open, who trades with who, and so on. Define a sequential markets equilibrium. Be explicit about the natural debt limit you are using.

4. State a proposition that establish the essential equivalence of the equilibrium concepts in the last two parts. Specify the relationships between the objects in the Arrow-Debreu economy and those in the sequential markets equilibrium.

5. For the Arrow-Debreu market structure, find the equilibrium prices and the consumption stream of each agent in equilibrium. Use the normalization $\hat{p}_0 = 1$.

6. Are the agents better off because of trade? Explain.

7. Prove that the equilibrium consumption sequences are constant over time. Graph the sequences of consumption sequences and prices. Rank the equilibrium discounted utilities $\{u(\hat{c}^i)\}_{i=\{1,2,3\}}$. Are they equal? Why is that so if the endowments are similar.

8. Price an asset that would give 0.05 units of consumption in each period.

9. Write the problem of a social planner who maximizes total welfare. Use λ_1 and λ_2 as the weights on agents 1 and 2.

10. Solve the social planner's problem. What conditions do we need on the planner's weights so that the planner's FOCs are identical to the competitive equilibrium FOCs?

11. Suppose now that the endowment process is

$$e_t^1 = \begin{cases} 3 & \text{if } t = \{0, 3, 6, \dots\} \\ 1 & \text{otherwise} \end{cases}$$

$$e_t^2 = \begin{cases} 3 & \text{if } t = \{1, 4, 7, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

$$e_t^3 = \begin{cases} 3 & \text{if } t = \{2, 5, 8, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

Are the consumption sequences constant? Why?

Problem 2 (From LS). *Suppose households 1 and 2 have one-period utility function $u(c^1)$ and $w(c^2)$, respectively, where u and w are both increasing, strictly concave, twice differentiable functions of a scalar consumption rate. Consider the Pareto problem*

$$v_\theta(c) = \max_{c^1, c^2} \theta u(c^1) + (1 - \theta) w(c^2)$$

subject to the constraint $c^1 + c^2 = c$.

1. Show that the solution of this problem has the form of a concave utility function $v_\theta(c)$, which depends on the the Pareto weight θ.

2. Show that $v'_\theta(c) = \theta u'(c^1) = (1 - \theta) w'(c^2)$.

The function $v_\theta(c)$ is the utility function of the representative consumer. Such a representative consumer always lurks within a complete markets competitive equilibrium even when preferences are heterogeneous. At a competitive equilibrium, the marginal utilities of the representative agent and every agent are proportional.

Problem 3. *Complete the steps to prove the first welfare theorem for the deterministic endowment economy seen in class:*

Proposition 1. *Let* $(\{\hat{c}_i^t\}_{t=0}^{\infty})_{i \in I}$ *be an Arrow-Debreu allocation for an endowment economy with associated prices* $\{\hat{p}_t\}_{t=0}^{\infty}$. *Then* $(\{\hat{c}_i^t\}_{t=0}^{\infty})_{i \in I}$ *is Pareto efficient.*

The proof will proceed by contradiction. Assume that $(\{\hat{c}_i^t\}_{t=0}^{\infty})_{i \in I}$ *is not Pareto efficient. Then, there exists another feasible allocation* $(\{\tilde{c}_i^t\}_{t=0}^{\infty})_{i \in I}$ *such that*

$$U(\tilde{c}^i) \geq U(\hat{c}^i) \text{ for all } i \in I$$
$$U(\tilde{c}^i) > U(\hat{c}^i) \text{ for at least one } i$$

Without loss of generality, assume that the strict inequality holds for $i = 1$.

1. Show that
$$\sum_{t=0}^{\infty} \hat{p}_t \tilde{c}_t^1 > \sum_{t=0}^{\infty} \hat{p}_t \hat{c}_t^1.$$

2. For all $i \neq 1$, show that
$$\sum_{t=0}^{\infty} \hat{p}_t \tilde{c}_t^i \geq \sum_{t=0}^{\infty} \hat{p}_t \hat{c}_t^i.$$

3. Complete the proof by combining the two intermediary results to find a contradiction.