1.

Exog states: $\{A_t\}$

Endog states: $\{N_t\}$

Jump / Controls: $\{C_t, V_t, Y_t, q_t, p_t, S_t\}$ ↗ (N_t is not a jump in this timing)

2. $$\max_{\{C_t, S_t, N_{t+1}\}} E_0 \sum \beta^t \Big\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \gamma_s S_t - \gamma_n N_t$$

$$+ \lambda_{1,t} \left(\Pi_t + W_t N_t - C_t \right)$$

$$+ \lambda_{2,t} \left((1-\delta_n) N_t + p_t S_t - N_{t+1} \right) \Big\}$$

FOC

C_t: $$C_t^{-\sigma} = \lambda_{1,t}$$

S_t: $$\gamma_s = p_t \lambda_{2,t}$$

N_{t+1}: $$\lambda_{2,t} = \beta E_t \left[\lambda_{1,t+1} W_{t+1} - \gamma_n + \lambda_{2,t+1} (1-\delta_n) \right]$$

3.

$$\max_{\{y_t, V_t, N_{t+1}\}} E_0 \sum \beta^t \Big\{ \frac{\lambda_{1,t}}{\lambda_{1,0}} \Big(\overbrace{y_t - W_t N_t - \phi V_t}^{\pi_t} \Big)$$

$$\theta_{1,t} \left(A_t N_t - y_t \right)$$

$$+ \theta_{2,t} \left((1-\sigma n) N_t + q_t V_t - N_{t+1} \right) \Big\}$$

FOC

y_t $$\frac{\lambda_{1,t}}{\lambda_{1,0}} = \theta_{1,t}$$

V_t $$\phi \frac{\lambda_{1,t}}{\lambda_{1,0}} = \theta_{2,t}\, q_t$$

N_{t+1} $$\theta_{2,t} = \beta E_t \left[\theta_{1,t+1} A_{t+1} - \frac{\lambda_{1,t+1}}{\lambda_{0,t+1}} W_{t+1} - \theta_{2,t+1} (1-\sigma n) \right]$$

combine

$$\frac{q_t}{\phi} \lambda_{1,t} = \beta E_t \left[\lambda_{1,t+1} \left(A_{t+1} - W_{t+1} \right) - (1-\sigma n) \frac{q_{t+1}}{\phi} \lambda_{t+1} \right] \quad \star$$

4. Planner's economy

$$V(N_t, A_t) = \max_{\{C_t, S_t, V_t, N_{t+1}\}} \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \gamma_s S_t - \gamma_n N_t$$

$$+ \beta E_t \left[V(N_{t+1}, A_{t+1}) \right]$$

s.t. (a) $N_{t+1} = (1-\delta) N_t + \chi V_t^{\varepsilon} S_t^{1-\varepsilon}$

(b) $C_t = A_t N_t - \phi V_t$

Since $V_t = \left(\dfrac{N_{t+1} - (1-\delta) N_t}{\chi S_t^{1-\varepsilon}} \right)^{\frac{1}{\varepsilon}}$ we have

$$V(N_t, A_t) = \max_{\{S_t, N_{t+1}\}} u(A_t N_t - \phi \square) - \gamma_s S_t - \gamma_n N_t$$

$$+ \beta E_t [V(N_{t+1}, A_{t+1}]$$

Since $V_t = \left(\dfrac{N_{t+1} - (1-\delta)N_t}{\chi S_t^{1-\varepsilon}}\right)^{\frac{1}{\varepsilon}}$ we have

$$V(N_t, A_t) = \max_{\{S_t, N_{t+1}\}} u(A_t N_t - \phi \square) - r_s S_t - \gamma_n N_t + \beta E_t[V(N_{t+1}, A_{t+1})]$$

copied from before

FOCs

$\boxed{S_t}$ $$-\gamma_s + u'(\,\cdot\,)\,\phi\,\frac{1}{\varepsilon}\overbrace{\left(\frac{N_{t+1} - (1-\delta)N_t}{\chi S_t^{1-\varepsilon}}\right)^{\frac{1}{\varepsilon}-1}\left(\frac{N_{t+1}-(1-\delta)N_t}{\chi}\right)(\varepsilon-1)\,S_t^{\varepsilon-2}}^{\text{can't simplify but don't need to}} = 0$$

$\boxed{N_{t+1}}$ $$u'(\;)\,\phi\,\frac{1}{\varepsilon}\left(\frac{N_{t+1} - (1-\delta)N_t}{\chi S_t^{1-\varepsilon}}\right)^{\frac{1}{\varepsilon}-1}\frac{1}{\chi S_t^{1-\varepsilon}}(-1)$$

$$+ \beta E_t V_1(N_{t+1}, A_{t+1}) = 0$$

But, by evelope condition, we have

$$V_1(\;) = u'(\;)\,\phi\,\frac{1}{\varepsilon}\left(\frac{N_{t+1} - (1-\delta)N_t}{\chi S_t^{1-\varepsilon}}\right)^{\frac{1}{\varepsilon}-1}\frac{1}{\chi S_t^{1-\varepsilon}}(1-\delta)(-1)^2$$

$+u'(\;)A_{t+1} \leftarrow$ conceptually this is the hard part, since diff't from before.

$$u'(\,)\,\frac{\phi}{\varepsilon}\left(\frac{N_{t+1}-(1-\delta)N_t}{\chi S_t^{1-\varepsilon}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\frac{1}{\chi S_t^{1-\varepsilon}}$$

$$=\beta E_t\left[u'(\,)\left[A_{t+1}+\frac{(1-\sigma)\phi}{\varepsilon}\left(\frac{N_{t+2}-(1-\delta)N_{t+1}}{\chi S_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\frac{1}{\chi S_{t+1}^{1-\varepsilon}}\right]\right]$$

5. Preferably, log linearize combined posting condition

$$\frac{q_t}{\phi}\lambda_{1,t} = \beta E_t\left[\lambda_{1,t+1}\left(A_{t+1} - w_{t+1}\right) + (1-\delta_n)\frac{q_{t+1}}{\phi}\lambda_{t+1}\right] \quad \star$$

but OK if not combined.

$$\frac{\exp(\hat{q}_t)}{\phi} = \beta E_t\left[\left(\frac{\exp(\hat{\lambda}_{t+1})}{\exp(\hat{\lambda}_t)}\right)\left(\exp(\hat{A}_{t+1}) - \exp(\hat{w}_{t+1}) - (1-\delta)\frac{\exp(\hat{q}_{t+1})}{\phi}\right)\right]$$

LHS:

$$\approx \frac{1}{\phi}\exp(\hat{q})(\hat{q}_t - \hat{q}) \quad \text{where } \hat{q} \equiv \log(q_{ss})$$

RHS

$$\approx E_t\left[\beta\left(A_{ss}(\hat{a}_{t+1} - \hat{a}) - w_{ss}(\hat{w}_{t+1} - \hat{w})\right) - \frac{(1-\delta)}{\phi}q_{ss}(\hat{q}_{t+1} - \hat{q})\right.$$

$$\left. + \beta\left(A_{ss} - w_{ss} - \frac{(1-\delta)q_{ss}}{\phi}\right)\exp(0)\left(\hat{\lambda}_{t+1} - \hat{\lambda}_t\right)\right]$$

since $\frac{\lambda_{t+1}}{\lambda_t} = 1$ in steady-state

6. We should use projection, this procedure captures both the non-linearities in the economy & the potential for stochastic outcomes. This method would approximate policy functions,

eg. $N_{t+1} = \sum a_n b_n(Z_t)$ where

b_m are basis functions,

a_n are weights on these functions

& $Z_t = \{A_t, N_t, \sigma_t\}$ are the states.

Then we search for $\{a_n\}$ to satisfy first order conditions.

Shooting: requires perfect foresight.

Linearization: no effects of risk, since as if risk neutral.

V.F.: slow & not needed since we have smooth choices.