Department of Economics
Cornell University

Instructor: Ryan Chahrour

Problem Set 4
Macroeconomics I
Due December 9, 2024

Recall the single-sector neoclassical model with search from class. In the first problem set, we solved this model using a linear approximation to the model around its non-stochastic steady state. In the second problem set, we solved the model using a shooting algorithm. In the third problem set, we solved the model using a value function iteration over a discretized state space. You should feel free to build on the example code shared on Canvas.

The parameters you should use in this problem set are the same as in Problem Set 1-3.

1. **Projection Method:**

   (a) Generate a script called `solve_projection`. The first part of the script should use our tools to generate the solution matrices g_x and h_x to the linearized (NOT log-linearized) model at the baseline parameters. This step is the same as before, so feel free to copy your code from the previous assignment.

   (b) Derive and report a formula that connects the unconditional standard deviations of $log(A)$ to ρ and σ_a. You should find that under our baseline calibration, `stda = 0.0320`. Using the `linspace` command, create a variable `agrid` that contains 7 equally-spaced values between minus and plus two standard deviations of $log(A_t)$ centered around zero.

   (c) Use `GH_Quadrature` to create a grid of 3 points for the productivity shock ϵ_{t+1}, and associated shock probabilities pw. Confirm that, using these values, $E[\epsilon_{t+1}^2] = \sigma_a^2$.

   (d) Using your results from (b) and (c) above, create a grid of potential future realization of $log(A_{t+1})$ by adding the possible grid values of the shocks ϵ_{t+1} to the possible grid values of $\rho \log(A_t)$. Call this 3×7 matrix `aprime`.

   (e) Using the `linspace` command, create a grid of 7 points for capital. The grid should be equally-spaced between $0.9 \times \bar{K}$ and $1.1 \times \bar{K}$. Similarly, create a grid of 21 points for employment. The grid should be equally-spaced between $0.8 \times \bar{N}$ and $1.2 \times \bar{N}$.

   (f) Now, use the command `ndgrid` to create a grid across all $7 \times 7 \times 21 \equiv N_s$ points in the state-space. Call the vector of values for productivity `aagr`, call the vector of values for capital `kkgr`, call the vector of value for employment `nngr`. Define `state_grid = [aagr;kkgr;nngr]`.

   (g) Using the linearized policy functions g_x and h_x and the grid you generated in part (c) above, compute initial guesses for the policy choices `kinit`, `ninit` for each point in the state space. Notice that the policy guesses in `kinit` and `ninit` generally lie between points on the policy grid.

(h) Create a function `residual(kinit,ninit,state_grid,allgrid,aprime,pw,param)`, with inputs listed as stated. Parts (i) - (m) refer to calculations you should do inside of the residual function.

(i) For each point in the state space, assume that N_t, K_{t+1} are chosen according to our initial guess. Using these values for policy and the model equations, compute I_t, V_t, and C_t.

(j) Now create a $3 \times N_s$ grid of possible future states. For possible future A_{t+1}, use `exp(agrid)`. For possible future states, K_{t+1} and N_t, use the same policy guesses used above.

(k) Now use the `ndim_simplex_eval` command to evaluate the future policy choices K_{t+2} and N_{t+1} at each of these potential future states. Using the value of the state and the policy at these points, use the model equations to compute I_{t+1}, V_{t+1}, and C_{t+1}.

(l) Use the future variables you computed above to evaluate the expectations terms from our equilibrium Euler equations

$$E_{1,t} \equiv E_t \left[\beta C_{t+1}^{-\sigma} \left(A_{t+1} \alpha \left(\frac{K_{t+1}}{N_{t+1}} \right)^{\alpha-1} + (1-\delta_k) \right) \right] \tag{1}$$

$$E_{1,t} \equiv E_t \left[\beta C_{t+1}^{-\sigma} \left(\frac{(1-\delta_n)\phi_n}{\varepsilon \chi V_{t+1}^{\varepsilon-1}} \right) \right] \tag{2}$$

You need to do this for each of the N_s current states today. To do this, you need sum all three of the possible future realizations starting from a given state today, weighted by the corresponding probability from pw.

(m) Using the terms compute in (l) above, let the output of your residual functions be the residuals from the two Euler equations of our model

$$R_1 \equiv C^{-\sigma} - E_{1,t} \tag{3}$$

$$R_2 \equiv C^{-\sigma} \left(\frac{\phi_n}{\varepsilon \chi V_t^{\varepsilon-1}} - A_t(1-\alpha)(K_t/N_t)^{\alpha} \right) - E_{2,t} \tag{4}$$

Your function should return a $2 \times N_s$ matrix of such residuals called `out`, with the first row containing the values of R_1 and the second row containing the values of R_2.

(n) Report the `sum(abs(out(:)))` associated with the initial policy function guesses based on the linear model solution. In my case, I find `sum(abs(out(:))) = 0.0341`.

(o) Using matlab's `fsolve` command, solve for the policy function values that zero out all of the residuals of your residual function. How many iterations fsolve needs to arrive at a solution? Report the value of your policy function for employment when A, K, and N are all simultaneously at the lowest values on their grid.

(p) Plot your policy functions for capital and labor choices against the values of the capital in the grid. Assume thatA_t and N_{t-1} are held constant at their steady-state values. Use a blue line for the initial linear policy functions and a green line for the policy functions you solved for in the non-linear problem. What do you notice about the solutions to that you have found?

(q) *Simulate 5000 periods the economy using the policy functions from the projection solution. Fill in the table below.

Table 1: Moments from simulated linear model

Moment	linear model value	projection solution value
Std log(Y)	0.0522	
Std log(C)		
Std log(I)	0.0975	
Std log(N)		