ECON 6130: Problem set 4

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Please upload your problem set on Canvas. You may work in groups, but you must turn in your own answers. Actively working on the assignments is *absolutely essential* for your understanding of the course material.

Problem 1. Consider the following problem:

$$\max_{\{c_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + k_{t+1} = e^{y_t} k_t^{\alpha} + (1 - \delta) k_t$$

where y_t is a random process.

1. Write this problem recursively (Bellman equation) assuming that y_t is a Markov chain. State conditions to guarantee the value function is continuous, monotone and concave. What are the state variable(s)? What are the control variable(s)?

2. Assume the following process for income:

   $$y_{t+1} = 0.98 y_t + \epsilon_t$$

   where ϵ_t is an iid normal shock with mean 0 and variance such that the long-run variance of y_t is 0.1. Construct a 7-point Markov chain approximation to this process. To do so you can use any commonly used technique. I recommend Tauchen (1986). Space approximation nodes between -3 and 3 standard deviations from the long run mean. Simulate your Markov chain. Compute it's long run mean, it's serial correlation and volatility.

3. Suppose preferences are log. Assume that $\beta = 0.95$, $\delta = 0.1$ and $\alpha = 0.35$. Compute the value function for this problem assuming a discrete grid of 100 equally spaced points for k. You need to pick appropriate bounds for this grid. Plot the value function as a function of k for all the values of the random process. Do the same for the policy function.

4. Simulate the model for a large number of periods and compute the standard deviations of (the log of) output, consumption and investment, and the correlations between output, consumption and investment. How does that compare to the data? (Hint: you should detrend the data using a method of your choice. All the data you need is on Fred. Make sure to use the "real", i.e. adjusted for inflation, time series and to take the log.)

Problem 2. Consider a neoclassical growth model with two sectors, one producing consumption goods and one producing investment goods. Consumption is given by $C_t = F(K_{Ct}, L_{Ct})$ and investment is given by $I_t = G(K_{It}, L_{It})$, where K_{jt} is the amount of capital in sector j at the beginning of period t and L_{jt} is the amount of labor used in sector j in period t. The total amount of labor in each period is equal to L (leisure is not valued by the household). Labor can be freely allocated in each period between the two sectors: $L = L_{Ct} + L_{It}$. Capital, by contrast, is sector-specific. Investment goods, however, can be used to augment the capital stock in either sector. In particular, the capital stocks in the two sectors evolve according to

$$K_{j,t+1} = (1 - \delta)K_{jt} + I_{jt}$$

where $I_t = I_{Ct} + I_{It}$.

The social planner seeks to maximize $\sum_{t=0}^{\infty} \beta^t u(C_t)$, given K_{C0} and K_{I0}, subject to the constraints on technology. Note that although leisure is not valued in the utility function the planner must nonetheless decide in each period how to allocate L across the two sectors.

1. Formulate the planner's optimization problem as a dynamic programming problem. What are the state variables? What are the choice variables? (Hint: you should have two of each.)
2. Find a set of first-order conditions and envelope conditions (using Benveniste-Scheinkman) that an optimal solution of the planning problem must satisfy.
3. Use your answer to part 2 to find a set of equations that determine the steady-state values of capital and labor (in each sector) in this economy.
4. Suppose that $F(K_{Ct}, L_{Ct}) = K_{Ct}^{\alpha} L_{Ct}^{1-\alpha}$ and $G(K_{It}, L_{It}) = K_{It}^{\gamma} L_{It}^{1-\gamma}$. Express the steady-state as a function of the structural parameters of the model.

Problem 3. We consider the neoclassical growth model with an externality. A representative firm produces output according to the production function

$$Y_t = F(K_t, N_t).$$

Capital depreciates at a constant rate $\delta \geq 0$. The aggregate resource constraint is

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t$$

where C_t is aggregate consumption.

There is a large number of identical households with total mass equal to 1. Each household is endowed with $k_0 = K_0$ units of capital and one unit of time every period. The household has preferences over identical consumption streams $\{c_t\}_{t=0}^{\infty}$ representable by the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t, C_t).$$

Note that the household chooses $\{c_t\}_{t=0}^{\infty}$, but takes aggregate consumption $\{C_t\}$ as given.

1. State the social planner problem recursively. Clearly identify the state and control variables.

2. Use the first-order condition and the envelope condition to derive the Euler equation of the social planner problem.

3. Define a recursive competitive equilibrium.

4. Use the first-order condition and the envelope condition to derive the Euler equation that the representative household faces.

5. For the purpose of this question, you can assume that there exists a unique competitive equilibrium. Is this equilibrium Pareto efficient? Yon don't need to provide a formal proof but use the answers to questions 2 and 4 to explain your answer. What is the intuition?