Problem Set 6

Due: TA Discussion, 11 October 2024.

1 Exercises from class notes

All from “4. Correspondences.pdf”.

Exercise 1. Give an example of a correspondence $F : X \rightrightarrows Y$ such that $F(x)$ is closed for some $x \in X$ but F is not closed at x.

Exercise 2. TFU: If a correspondence $F : X \rightrightarrows Y$ is upper hemi-continuous, then $F(x)$ is closed for every $x \in X$.

Exercise 3. Are following correspondences are upper hemi-continuous and/or lower hemi-continuous?

$$F(x) = \begin{cases} \{4 - x, 2 - x\}, & \text{if } x < 2, \\ [2 - x, 4 - x], & \text{if } 2 \leq x \leq 3, \\ \{x - 3\}, & \text{if } x > 3. \end{cases}$$

$$G(x) = \begin{cases} \{4 - x, 2 - x\}, & \text{if } x < 2, \\ [3 - x, 5 - x], & \text{if } 2 \leq x \leq 3, \\ \{x - 3\}, & \text{if } x > 3. \end{cases}$$

Exercise 4. Prove that the budget correspondence, $\Gamma(\mathbf{p}, m) : \mathbb{R}^{d+1}_{++} \rightrightarrows \mathbb{R}^d$ such that

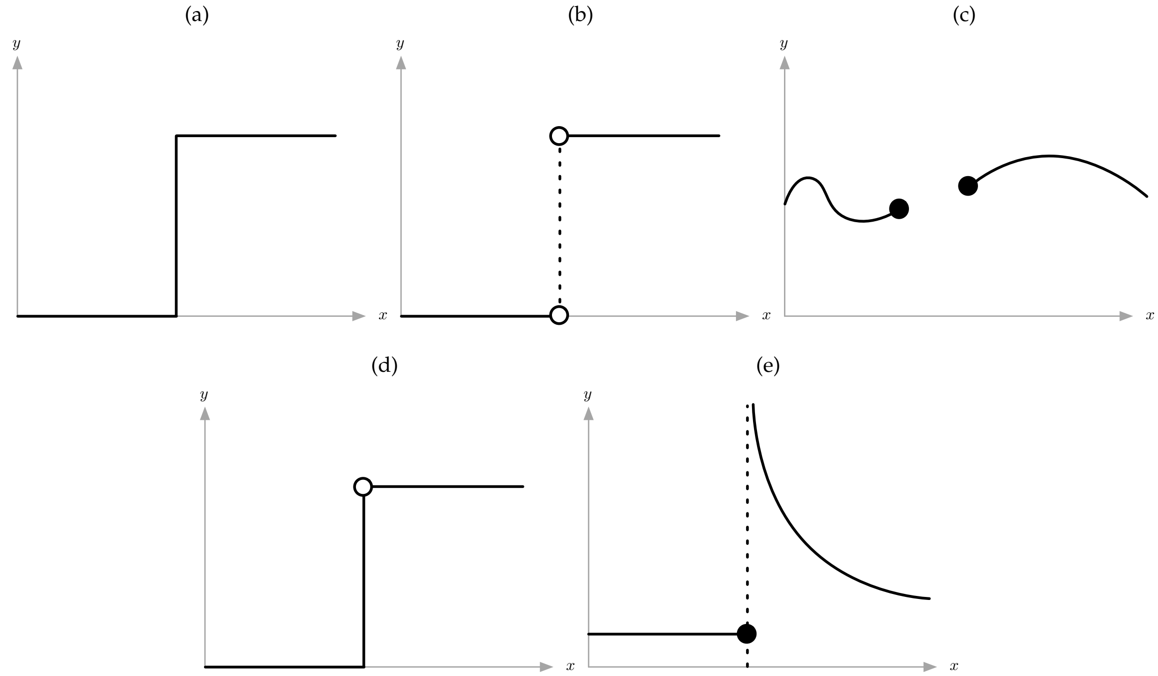
$$\Gamma(\mathbf{p}, m) := \left\{ \mathbf{x} \in \mathbb{R}^d_+ : \mathbf{p} \cdot \mathbf{x} \leq m \right\},$$

is continuous. What does the Berge’s theorem of the maximum tell you about the consumer’s problem when the agent’s utility function is continuous?

2 Additional Exercises

State whether each of the following correspondence (that is a mapping from $\mathbb{R}$ to $\mathbb{R}$) is upper hemi-continuous and/or lower hemi-continuous, as well whether the correspondence has a closed graph.

Figure 1: Additional Exercises



State whether the following correspondences (that is a mapping from $\mathbb{R}$ to $\mathbb{R}$) are upper hemi-continuous and/or lower hemi-continuous at x_1 and x_2. Are any of them upper hemi-continuous and/or lower hemi-continuous?

Figure 2: Additional Exercises