Econ 6190 Problem Set 10

Fall 2024

1. Take the model $X \sim \mathrm{N}(\mu, 4)$. A sample of $n = 16$ independent realizations of X was collected, and the sample mean $\bar{X} = 20.5$. A sample of $n = 16$ independent realizations of X was collected, and the sample mean $\bar{X} = 20.5$. Find a 98% confidence interval for μ.

2. [Hansen 14.4] You have the point estimate $\hat{\theta} = 0.45$ and standard errors $s(\hat{\theta}) = 0.28$. You are interested in $\beta = \exp(\theta)$.

   (a) Find $\hat{\beta}$.

   (b) Use the delta method to find a standard error $s(\hat{\beta})$.

   (c) Use the above to calculate a 95% asymptotic confidence interval for $\hat{\beta}$.

   (d) Calculate a 95% asymptotic confidence interval $[L, U]$ for the original parameter θ. Calculate a 95% asymptotic confidence interval for β as $[\exp(L), \exp(U)]$. Can you explain why this is a valid choice? Compare this interval with your answer in (c).

3. [Hansen] Answer the following questions.

   (a) A confidence interval for the mean of a variable X is $[L, U]$. You decided to rescale your data, so set $Y = \frac{X}{1000}$. Find the confidence interval for the mean of Y.

   (b) In general, let $C = [L, U]$ be a $1-\alpha$ confidence interval for θ. Consider $\beta = h(\theta)$ where $h(\theta)$ is mono tonically increasing. Set $C_\beta = [h(L), h(U)]$. Evaluate the converage probability of C_β for β. Is C_β a $1-\alpha$ confidence interval?

4. Let the random variable X be normally distributed with mean μ and variance 1. You are given a random sample of 16 observations.

   (a) Construct a one sided 95% confidence interval for μ that has form $[\hat{L}, \infty)$ for some statistic $\hat{L}$.

   (b) Construct a two sided 95% confidence interval for μ.

   (c) Show that the rejection of the null $\mathbb{H}_0 : \mu = 0$ against $\mathbb{H}_1 : \mu \neq 0$ with size 5% based on t test corresponds to the rejection of $\mathbb{H}_0 : \mu = 0$ when zero does not lie in the 95% confidence interval for μ constructed in part (b).

   (d) How would your answers be affected when you would not have known the variance of the random variable?

(e) How would your answers be affected when you would not have known the variance of the random variable but the sample size is 100?

5. [Hansen] Answer the following questions.

   (a) A confidence interval for the mean of a variable X is $[L, U]$. You decided to rescale your data, so set $Y = \frac{X}{1000}$. Find the confidence interval for the mean of Y.

   (b) In general, let $C = [L, U]$ be a $1-\alpha$ confidence interval for θ. Consider $\beta = h(\theta)$ where $h(\theta)$ is monotonically increasing. Set $C_\beta = [h(L), h(U)]$. Evaluate the converge probability of C_β for β. Is C_β a $1-\alpha$ confidence interval?

6. [Hansen 14.7] A friend suggests the following confidence interval for θ: they draw a random number $U \sim U[0, 1]$ and set

$$C = \begin{cases} \mathbb{R} & \text{if } U \leq 0.95 \\ \emptyset & \text{if } U > 0.95 \end{cases}.$$

   (a) What is the coverage probability of C?

   (b) Is C a good choice for a confidence interval? Explain.