ECON 6090 - TA Section 1

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Exercises

Rational Preference Relations

1. Suppose $\succsim$ is a rational preference relation on a set X. Let x and y be elements of X. We define equivalence, denoted $x \sim y$, by $x \succsim y$ and $y \succsim x$. Is $\sim$ transitive?

2. Are the following preference relations $\succsim$ rational?

   (a) Let $\succsim$ be defined on $\mathbb{R}$ by: $y \succsim x$ iff $y \geq x + \varepsilon, \varepsilon$ is a positive number.

   (b) Let $\succsim$ be defined on $\mathbb{R}$ by: $y \succsim x$ iff $y \geq x - \varepsilon, \varepsilon$ is a positive number.

   (c) $X = \{a, b, c\}. C^*(\{a, b\}, \succsim) = \{b\}. C^*(\{b, c\}, \succsim) = \{c\}. C^*(\{a, b, c\}, \succsim) = \{c\}$.

   (d) Agents 1 and 2 are facing the same choice set X. Agent 1 has a rational preference relation $\succsim_1$, consumer 2 's preference relation is given by $\succsim_2 := \succ_1$. Is consumer 2 's preference rational?

   (e) Consider the lexicographic preference relation $\succsim$ on $\mathbb{R}^2_+ : (x_1, x_2) \succsim (y_1, y_2)$ if and only if $x_1 > y_1$ or $x_1 = y_1$ and $x_2 \geq y_2$. Is $\succsim$ a rational preference relation?

3. (2022 Q)

   (a) Consider a finite set of alternatives $A = \{a_1, a_2, \ldots, a_N\}$ and a decision maker with rational preferences $\geq$ on A. An alternative $a^* \in A$ is said to be a best alternative if $a^* \geq a_i$ for all $a_i \in A$. Show that a best alternative exists for this decision maker and this set of alternatives A.

   (b) Consider the set of alternatives $A' = \{a_1, a_2, \ldots, a_{N-1}\}$; A with the alternative a_N deleted. Let a' be a best alternative in the set A' for the decision maker from part (a) with preferences $\geq$. Show that $a^* \geq a'$. We can interpret this result as showing that deleting an alternative from a finite set of alternatives cannot make a decision maker with rational preferences better off.

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Choice rules

1. (2008 Final) Let the set of alternatives be $X = \{a, b, c, d\}$ and let B be the set of all nonempty subsets of X. Suppose we have a choice function such that $C(\{a, b, d\}) = \{a, b\}$ and $C(\{a, b, c\}) = \{b\}$. Is there a rational preference relation $\succsim$ on X such that $C(\cdot) = C^*(\cdot, \succsim)$ for all elements in B and $C^*(., \succsim)$ is consistent with the information given about $C(\cdot)$? Explain briefly.

2. Suppose instead of the data above, we have a choice function such that $C(\{a, b, c\}) = \{a, b\}, C(\{b, c\}) = \{b\}, C(\{c, d\}) = \{c\}$ and $C(\{a, d\}) = \{a, d\}$. Is there a rational preference relation $\succsim$ on X such that $C(\cdot) = C^*(\cdot, \succsim)$ for all elements in B and $C^*(\cdot, \succsim)$ is consistent with the information given about $C(\cdot)$? Explain briefly.