ECON 6090

Problem Set 6

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1. Monopolists and quality

   (a) In this case, where there is full information, the monopolist can choose a different quality level for each consumer type θ, and maximize profit for that consumer type. Fix some $\theta > 0$. The consumer's utility from purchasing a good of quality s at price T is

   $$u(s, T; \theta) = s \cdot \theta - T$$

   and the monopolist's profit is

   $$\pi_\theta(s, T) = T - 5s^2$$

   Note that if we were being precise here, these would be defined piecewise, if the consumer purchases the good or not. This is not necessary now, as we will show that the consumer θ will always purchase the good. The monopolist will ensure this by ensuring that $s \cdot \theta - T \geq 0$. In fact, to maximize profit, we will have that $s \cdot \theta - T = 0 \Longrightarrow T = s \cdot \theta$. This implies that profit is

   $$\pi_\theta(s, T) = s \cdot \theta - 5s^2$$

   Taking first order conditions, we get that the optimal quality is where

   $$\theta - 10s = 0 \Longrightarrow s^\star = \frac{\theta}{10}$$

   Substituting back, we get that the optimal price is

   $$T^\star = \theta \cdot \frac{\theta}{10} = \frac{\theta^2}{10}$$

   The monopolist will extract all of the surplus of each consumer, so $u(s^\star, T^\star; \theta) = 0$ for all θ, and for a certain θ,

   $$\pi_\theta(s^\star, t^\star) = T^\star - 5(s^\star)^2 = \frac{\theta^2}{10} - \frac{\theta^2}{20} = \frac{\theta^2}{20}$$

   (b) Now, we suppose that the seller cannot observe θ. We will assume that they produce goods of two qualities, s_H and s_L, designated for each type of θ, and priced at T_H, T_L. We will need the classic two constraints to hold: individual rationality and incentive compatibility. First, for the choice of each type of consumer to purchase to be rational, we need that

   $$\theta_L s_L - T_L \geq 0 \qquad \text{and} \qquad \theta_H s_H - T_H \geq 0$$

   Next, for the consumers to want their designated good rather than the other's, we need that

   $$\theta_L s_L - T_L \geq \theta_L s_H - T_H \qquad \text{and} \qquad \theta_H s_H - T_H \geq \theta_H s_L - T_L$$

   The monopolist maximizes profit subject to these constraints, where expected profit is

   $$\Pi = \beta(T_L - 5s_L^2) + (1 - \beta)(T_H - 5s_H^2)$$

We will make the standard assumptions that the individual rationality constraint for the low type and the incentive compatibility constraint for the high type each hold with equality. This implies that

$$T_L = \theta_L s_L \quad \text{and} \quad T_H = \theta_H(s_H - s_L) + \theta_L s_L$$

Substituting, the monopolist's profit becomes

$$\Pi = \beta(\theta_L s_L - 5s_L^2) + (1-\beta)(\theta_H(s_H - s_L) + \theta_L s_L - 5s_H^2)$$

To find the optimal choice of s_L and s_H, we take first order conditions, and get that

$$\frac{\partial \Pi}{\partial s_L} = \beta(\theta_L - 10s_L) + (1-\beta)(\theta_L - \theta_H) = 0 \Longrightarrow s_L^\star = \frac{\theta_L - (1-\beta)\theta_H}{10\beta}$$

and

$$\frac{\partial \Pi}{\partial s_H} = (1-\beta)\theta_H - (1-\beta)10s_H = 0 \Longrightarrow s_H^\star = \frac{\theta_H}{10}$$

From the binding constraints, we can calculate the optimal prices. We have that

$$T_L^\star = \theta_L \cdot \frac{\theta_L - (1-\beta)\theta_H}{10\beta} = \frac{\theta_L^2 - (1-\beta)\theta_H\theta_L}{10\beta}$$

and

$$T_H^\star = \theta_H\left(\frac{\theta_H}{10} - \frac{\theta_L - (1-\beta)\theta_H}{10\beta}\right) + \frac{\theta_L^2 - (1-\beta)\theta_H\theta_L}{10\beta} = \frac{\theta_H^2 - (2-\beta)\theta_H\theta_L + \theta_L^2}{10\beta}$$

Note that these solutions admit a corner. We have that the fully described solution is:

$$s_L^\star = \begin{cases} \frac{\theta_L - (1-\beta)\theta_H}{10\beta} & \theta_L \geq (1-\beta)\theta_H \\ 0 & \text{otherwise} \end{cases}$$

$$s_H^\star = \begin{cases} \frac{\theta_H}{10} & \theta_H^2 + \theta_L^2 \geq (2-\beta)\theta_H\theta_L \\ 0 & \text{otherwise} \end{cases}$$

$$T_L^\star = \begin{cases} \frac{\theta_L^2 - (1-\beta)\theta_H\theta_L}{10\beta} & \theta_L \geq (1-\beta)\theta_H \\ 0 & \text{otherwise} \end{cases}$$

$$T_H^\star = \begin{cases} \frac{\theta_H^2 - (2-\beta)\theta_H\theta_L + \theta_L^2}{10\beta} & \theta_H^2 + \theta_L^2 \geq (2-\beta)\theta_H\theta_L \\ 0 & \text{otherwise} \end{cases}$$

The informational rents are as follows. Since the low types' individual rationality constraint binds, they will attain no utility in equilibrium, which is the same as in the full information case. The high types, meanwhile, will (when we are not in the corner) attain utility:

$$u(s_H^\star \mid \theta_H) = \theta_H \cdot s_H^\star - T_H^\star = \frac{\theta_H^2}{10} - \frac{\theta_H^2 - (2-\beta)\theta_H\theta_L + \theta_L^2}{10\beta} = \frac{\theta_L^2 - (1-\beta)\theta_H^2 - (2-\beta)\theta_L\theta_H}{10\beta}$$

This is the informational rent the monopolist pays.

2. Constructing a bridge

   (a) The government is attempting to minimize the cost, subject to the monopolist attaining their reservation utility. They will ensure that the monopolist's utility is exactly equal to their reservation, which functions as in individual rationality constraint. We need that

$$\bar{u} = t - \frac{e^2}{2} = t - \frac{e^2}{2} \Longrightarrow t = \bar{u} + \frac{e^2}{2}$$

The government is minimizing the total cost, which is the cost function $t + c$, subject to the constraint. Recalling that $c = \theta - e$, we get that they minimize

$$C = \bar{u} + \frac{e^2}{2} + \theta - e$$

Taking first order conditions to find the ideal induced effort, we get that

$$\frac{\partial C}{\partial e} = e - 1 = 0 \Longrightarrow e^{FB} = 1$$

The transfer that induces e^{FB} is $t^{FB} = \bar{u} + \frac{1}{2}$. Note that the costs are different for each type. We have that $C^{FB}(5) = \bar{u} + 4.5$, and $C^{FB}(8) = \bar{u} + 7.5$.

(b) (Not able to figure out)

3. MWG 13.C.5

(a) Since the consumer is risk-neutral, they will buy the product if and only if its expected valuation is greater than its price. Mathematically, they will buy if and only if

$$p \leq \lambda v_H + (1 - \lambda)v_L$$

(b) There can be no separating equilibrium. To see why, assume that we do have a separating equilibrium, in which high-quality firms spend A on advertising and low-quality firms spend 0. Low-quality firms will spend 0 because in a separating equilibrium they will be immediately identified, so it is better to spend nothing than to advertise at all, since because $p > v_L$, nobody will buy their product. For incentive compatibility to hold for the high quality firms, it must be the case that $p - c_H - A > 0$. However, since $c_L < c_H$, in this case it would improve a low-quality firm's outcomes to deviate and spend A pretending to be a high-quality firm, since $p - c_L - A > p - c_H - A > 0$. Thus, there will be deviation, so this is not a separating equilibrium.

4. MWG 13.C.6

(a) Note first that since banks are risk-neutral and competitive, the equilibrium level of R will be the actuarily fair level of R. The expected payout to the bank for a funded project will be equal to $(1 + r)$, the cost to fund a project. For an arbitrary project, funded at rate R, the expected payout means that

$$\lambda\left(p_G R + (1 - p_G)0\right) + (1 - \lambda)\left(p_B R + (1 - p_B)0\right) = 1 + r$$

which implies that

$$\lambda p_G R + (1 - \lambda)p_B R = 1 + r \Longrightarrow R = \frac{1 + r}{\lambda p_G + (1 - \lambda)p_B}$$

A good entrepreneur will pursue a project if

$$p_G(\Pi - R) \geq 0 \Longrightarrow \Pi \geq \frac{1 + r}{\lambda p_G + (1 - \lambda)p_B}$$

and a bad entrepreneur will pursue a project if

$$p_B(\Pi - R) \geq 0 \Longrightarrow \Pi \geq \frac{1 + r}{\lambda p_G + (1 - \lambda)p_B}$$

Thus, if R is low enough (meaning if the proportion of good projects is high enough), every entrepreneur's project will be both funded and pursued. If not, nobody's project will go forward.

(b) Suppose that the entrepreneur can contribute some of their own funds.

i. A good entrepreneur's expected payout is

$$p_G(\Pi - (1-x)R) + (1-p_G)0 - x(1+\rho) = p_G(\Pi - R) + x(p_G R - (1+\rho))$$

A bad entrepreneur's expected payout is

$$p_B(\Pi - (1-x)R) + (1-p_B)0 - x(1+\rho) = p_B(\Pi - R) + x(p_B R - (1+\rho))$$

ii. Note first that since the rate that entrepreneurs borrow money is higher than banks, the best separating Bayesian equilibrium will be the one with the minimal x that separates the good and bad entrepreneurs. In a separating equilibrium, banks will offer good entrepreneurs the actuarily fair rate $R = \frac{1+r}{p_G} < \Pi$, and will offer bad entrepreneurs the actuarily fair rate $R = \frac{1+r}{p_B} > \Pi$. In this equilibrium, no bad entrepreneurs will accept this rate. We can find x by identifying the level above which a bad entrepreneur will attain negative utility if offered the good entrepreneurs' rate (rather than 0 from offering $x = 0$ and not pursuing the project). This will be the case if

$$p_B\left(\Pi - \frac{1+r}{p_G}\right) + x\left(p_B\frac{1+r}{p_G} - (1+\rho)\right) = 0$$

This implies that

$$x^\star = \frac{p_B\Pi - p_B\frac{1+r}{p_G}}{(1+\rho) - p_B\frac{1+r}{p_G}} = \frac{p_G p_B\Pi - p_B(1+r)}{p_G(1+\rho) - p_B(1+r)}$$

which is less than 1 since $p_B\Pi < 1+r < 1+\rho$. Thus, our perfect separating Bayesian equilibrium is:

Bad entrepreneurs will contribute $x = 0$ and accept the bank's offer if $R \leq \Pi$. Good entrepreneurs will contribute $x = \frac{p_G p_B\Pi - p_B(1+r)}{p_G(1+\rho) - p_B(1+r)} = x^\star$ and accept the bank's offer if $R \leq \frac{1+r}{p_G}$. The bank will offer $R = \frac{1+r}{p_B}$ if the entrepreneur contributes 0 and $R = \frac{1+r}{p_G}$ if the entrepreneur contributes $x^\star$. All good projects will be funded, all bad projects will be abandoned.

iii. Bad entrepreneurs will be (weakly) worse off in the separating equilibrium, since they were sometimes funded previously (depending on λ) and are never funded now. Good entrepreneurs will be better off for small λ, since they now get funded, but worse off for large λ since contributing their own funds is costly.