Introduction

The Economics of Asymmetric Information and Contracts

ECON 6090

This section provides an introduction to the economics of:

- asymmetric information
- contracts and mechanism design.

We say that we have **complete information** if all agents know all the relevant information.

We say that information is **incomplete** otherwise:

- symmetric incomplete information: some variables are unknown, but no player has privileged information;
- asymmetric, some players have more information than others.

Information is asymmetric if it is incomplete and agents have different information sets.

Markets are often characterized by asymmetric information

For example:

- buy a car, the seller knows a lot more than you about its quality;
- insurance, you know a lot more about your health or driving habits;
- an entrepreneur, knows about his/her habits and the quality of the project.

We have two broad categories of asymmetric information problems.

Adverse selection: when the asymmetric information concerns the characteristics of some agent:

- insurance,
- lending,
- selling

Moral hazard: when the asymmetric information concerns the action of some player:

- work relations
- insurance
- lending

What are the relevant questions?

How do uninformed agents react to their condition?

- stay out of the market
- enter the market and ignore the asymmetry
- "control" the market (i.e. design a contract) to mitigate asymmetry

How do informed agents react?

What are the general implications for markets?

Demand and supply with informational asymmetries

Market failure: Akerlof's model

Consider a labor market in which a worker produces θ units.

θ has distribution $F(\theta)$ in $[\underline{\theta}, \overline{\theta}]$ with $0 \leq \underline{\theta} < \overline{\theta} < \infty$

Firms hire workers to produce the good and sell it in a competitive market at price $p = 1$.

The number of workers is N.

Firms are risk neutral.

Workers have a reservation value for their time: $r(\theta)$.

If a worker of type θ does not work, s/he will obtain $r(\theta)$.

Complete information benchmark

In a competitive equilibrium with complete information:

- All workers with $r(\theta) < \theta$ are employed.
- $w(\theta) = \theta$ for the workers that are employed.
- $w(\theta) < \theta$, for the unemployed.

This market outcome is **Pareto optimal**: it is not possible to make any agent strictly better off without making some agent strictly worse off.

Aggregate surplus is:

$$W^* = \int_{\underline{\theta}}^{\overline{\theta}} N[1_\theta \cdot \theta + (1 - 1_\theta)r(\theta)]dF(\theta)$$

This aggregate surplus is clearly maximized by setting $1_\theta = 1$ iff $r(\theta) < \theta$ and 0 otherwise.

Equilibrium with asymmetric information

Workers types are unobservable, so in this market we can have only one price: w

(as opposed to the case with complete info, where we had a market for each θ)

Supply in this market: $\Theta(w) = \left\{\theta \text{ st. } r(\theta) \le w\right\}$, so

$$S(w) = F(r^{-1}(w))$$

For simplicity we assume that indifferent workers choose to work.

Demand is:

$$D(w) = \begin{cases} 0 & E\theta < w \\ [0, \infty] & E\theta = w \\ \infty & E\theta > w \end{cases}$$

It is clear that a market equilibrium w such that $D(w) = S(w)$, must be such that $w = E\theta$

At the same time, $E\theta$ must be consistent with supply, so:

$$w = E[\theta; r(\theta) \leq w]$$

This condition is called *rational expectations*.

Summarizing:

Definition. *In a competitive market model with unobservable worker's productivity, a competitive equilibrium is a wage rate* w^* *and a set of workers* Θ^* *such that:*

$$\Theta^* = \left\{\theta \text{ s.t. } r(\theta) \leq w\right\}$$

and:

$$w^* = E[\theta; \theta \in \Theta^*]$$

Comment: The rational expectation requirement is well defined only if Θ^* is non empty.

If Θ^* is an empty set, we need to specify "out of equilibrium" beliefs, since the firms expects no supply of labor.

For now we assume that if Θ^* is empty, then $w^* = E\theta$, the unconditional expectation.

At this stage, this assumption is as good as any other.

Inefficiency due to incomplete information

In general, with imperfect information, a competitive equilibrium is Pareto inefficient.

To see this point, assume $r(\theta) = r$, so constant.

The Pareto optimal allocation requires all workers with $\theta > r$ work, and all types with $\theta < r$ not work.

But this is impossible in a competitive equilibrium.

- If $w > r$, all types work
- If $w < r$, no type works

- If $w = r$, then types are indifferent, but there is no guarantee that they sort out as required.

The problem is that firms are unable to distinguish types, and the single price is not enough for the workers to sort themselves out.

Adverse selection and Market unraveling

Let us now consider the more realistic case in which $r(\theta)$ is increasing in θ.

This means that more productive types have better options.

This leads to the phenomenon of *Adverse selection*.

To keep the analysis simple let us assume:

- $r(\theta) \leq \theta$ for all θ, so it is efficient to have full employment.
- $r(\theta)$ is strictly increasing in θ

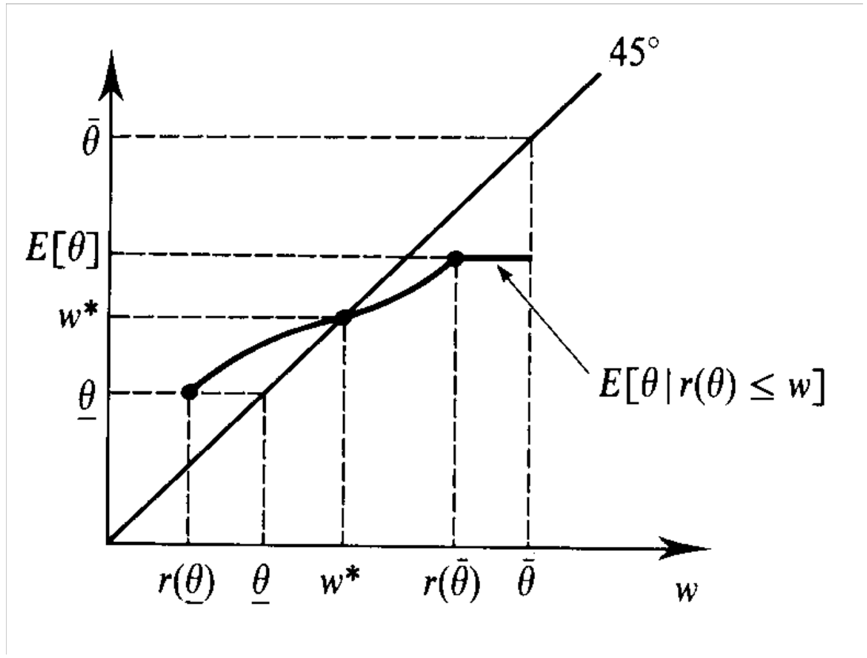
Now we have that $E\left[\theta \text{ st. } r(\theta) \leq w\right]$ is continuous in w (if F has a density f), increasing in w.

Note that:

- $E[\theta \text{ s.t. } r(\theta) \leq r(\underline{\theta})] = \underline{\theta} \geq r(\underline{\theta})$,

- and $E[\theta \text{ s.t. } r(\theta) \leq r(\overline{\theta})] = E\theta < \overline{\theta}$

So we have this figure...


45°
$\bar{\theta}$
$E[\theta]$
w^*
$\underline{\theta}$
$E[\theta \mid r(\theta) \leq w]$
$r(\underline{\theta})$
$\underline{\theta}$
w^*
$r(\bar{\theta})$
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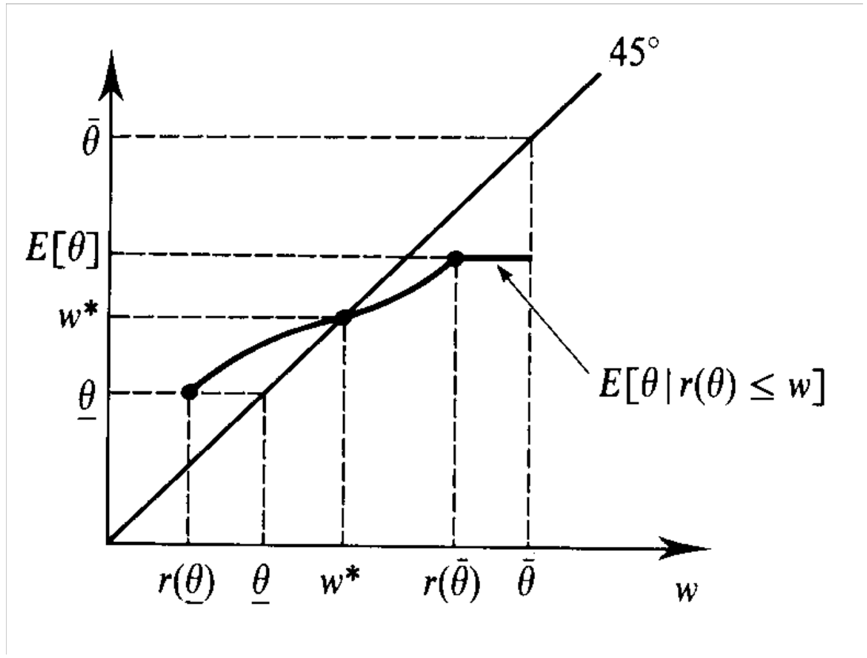
w



where $E\big[\theta$ s.t. $r(\theta) \le w\big]$ is above the 45^o line at $w = r(\underline{\theta})$; and below at $w = r(\overline{\theta})$.

We must have at least a $\omega^* \in (\underline{\theta}, \overline{\theta})$ such that:

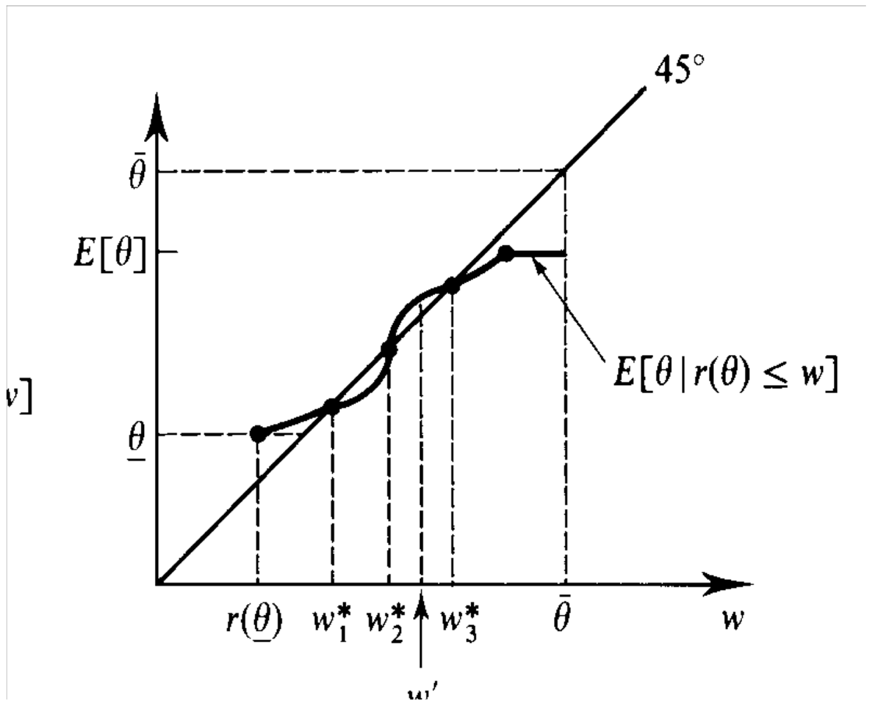
$$\omega^* = E\Big[\theta \text{ s.t. } r(\theta) \leq \omega^*)\Big]$$

This characterization immediately proves that the equilibrium is inefficient:

- It would be optimal to have all types employed;
- but only types $\theta \leq r^{-1}(\omega^*) < \overline{\theta}$, are indeed employed.

Two comments

First, we may have multiple equilibria



Equilibria are Pareto ranked:

- in correspondence to the w_i^*s, profits are constant at zero,
- but workers' welfare is increasing in w_i^*.

Second, the market may totally collapse (which is the classic point made by Akerlof).

For instance, assume $r(\theta) = \alpha\theta$ with $\alpha < 1$

θ is uniform in $[\underline{\theta}, \overline{\theta}] = [0, 2]$

In this case:

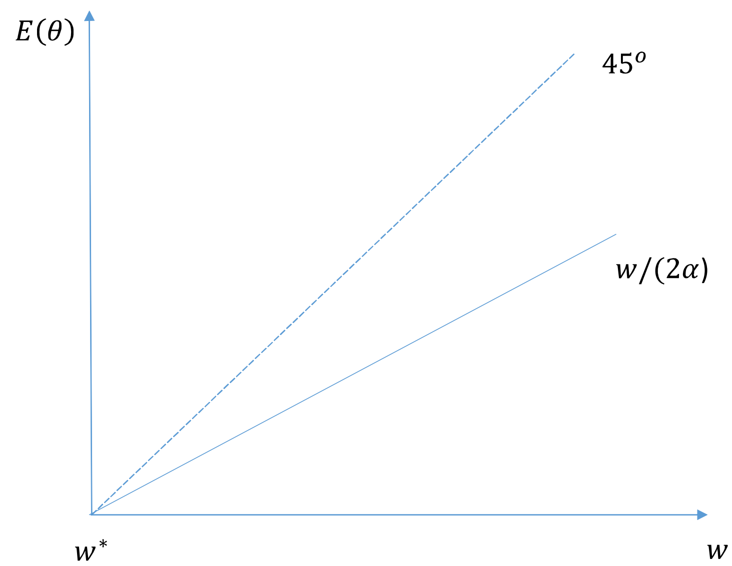
$$E\Big[\theta \text{ s.t. } r(\theta) \leq w)\Big] = E\Big[\theta \text{ s.t. } \alpha\theta \leq w)\Big]$$

$$= E\Big[\theta \text{ s.t. } \theta \leq \frac{w}{\alpha})\Big] = \frac{w}{2\alpha}$$

In this case, when $\alpha > 1/2$, the market collapses to zero.

Indeed, when $\alpha > 1/2$, the only fixed point is:

$$\omega^* = 0 = E\Big[\theta \text{ s.t. } r(\theta) \le 0)\Big]$$



Can we fix it with public intervention?

Can the government intervene and fix the problems?

A case in which it is possible is when there are multiple equilibria.

In this case the government could shift the equilibrium to $w^* = \max w_i^*$.

Can the government do better?

Surely if the government could see the types, but this is implausible.

A *Constrained Pareto Optimum* is a Pareto Optimum achievable by a planner with no informational advantage.

Is there a Constarined Pareto Optimum that is better than the best competitive equilibrium?

The answer is no.

The planner chooses w_e and w_u

Given this all workers of type $\theta \leq \widehat{\theta}$ will work, where:

$$w_u + r\left(\widehat{\theta}\right) = w_e \qquad \text{T}$$

So the government can only choose a $\widehat{\theta}$, w_e and w_u such that (T) and budget balance is satisfied:

$$w_e F(\widehat{\theta}) + w_u\left[1 - F(\widehat{\theta})\right] \leq \int \theta dF(\theta) \qquad \text{B}$$

Substitute (T) in (B), and we get:

$$w_u(\widehat{\theta}) = \int \theta dF(\theta) - r\left(\widehat{\theta}\right)F\left(\widehat{\theta}\right)$$

$$w_e(\widehat{\theta}) = \int \theta dF(\theta) + r\left(\widehat{\theta}\right)\left[1 - F\left(\widehat{\theta}\right)\right]$$

That is:

$$w_u(\widehat{\theta}) = F\left(\widehat{\theta}\right)\left[E\left[\theta;\theta \le \widehat{\theta}\right] - r\left(\widehat{\theta}\right)\right] \qquad \text{W}$$

$$w_e(\widehat{\theta}) = F\left(\widehat{\theta}\right)\left[E\left[\theta;\theta \le \widehat{\theta}\right] - r\left(\widehat{\theta}\right)\right] + r\left(\widehat{\theta}\right)$$

Let θ^* be the highest type employed in the highest competitive equilibrium, so:

$$r(\theta^*) = E[\theta; \theta \leq \theta^*] = w^*$$

If the government selects $\widehat{\theta} = \theta^*$, we have $w_e(\widehat{\theta}) = w^*$, $w_u(\widehat{\theta}) = 0$. So the outcome is the competitive equilibrium.

There are two other possibilities: $\widehat{\theta} > \theta^*$ and $\widehat{\theta} < \theta^*$.

If $\widehat{\theta} < \theta^*$, we have:

$$w_e(\widehat{\theta}) = F\left(\widehat{\theta}\right)\left[E\left[\theta;\theta \leq \widehat{\theta}\right] - r\left(\widehat{\theta}\right)\right] + r\left(\widehat{\theta}\right)$$

$$< F\left(\widehat{\theta}\right)\left[E\left[\theta;\theta \leq \widehat{\theta}\right] - r(\theta^*)\right] + r(\theta^*)$$

since $r(\theta^*) > r\left(\widehat{\theta}\right)$.

So:

$$w_e(\widehat{\theta}) - r(\theta^*) \leq F\left(\widehat{\theta}\right)\left[E\left[\theta;\theta \leq \widehat{\theta}\,\right] - r(\theta^*)\right]$$

$$= F\left(\widehat{\theta}\right)\left[E\left[\theta;\theta \leq \widehat{\theta}\,\right] - E[\theta;\theta \leq \theta^*]\right] < 0$$

It follows that:

$$w_e(\widehat{\theta}) < r(\theta^*) = w^*,$$

Low types were working in the competitive equilibrium for a higher wage, now they are worse off.

Assume now $\widehat{\theta} > \theta^*$. We must have $E[\theta; r(\theta) \leq w] < w$ for all $w \geq w^*$, else w^* would not be the highest competitive equilibrium.

Since $w^* = r(\theta^*)$ and $r(\theta)$ is increasing, $r\left(\widehat{\theta}\right) > r(\theta^*) = w^*$, so:

$$E\left[\theta; r(\theta) \leq r\left(\widehat{\theta}\right)\right] < r\left(\widehat{\theta}\right)$$

for $\widehat{\theta} \geq \theta^*$.

So $w_u(\widehat{\theta}) = F\left(\widehat{\theta}\right)\left[E\left[\theta; \theta \leq \widehat{\theta}\right] - r\left(\widehat{\theta}\right)\right] < 0$, implying that high types who were and remain unemployed are worse off now.

The market strikes back I

Signalling

One way the market may bypass the information asymmetry is by allowing workers to signal their type.

Assume here that there are two types $0 < \theta_L < \theta_H$, with $\Pr(\theta_H) = \lambda$.

We now assume that workers can get some education e.

To make the point more striking, education is unproductive.

The cost of education is $C(e,\theta)$ with:

- Usual technological assumptions: $C(0,\theta) = 0$, $C_e(e,\theta) > 0$, $C_{ee}(e,\theta) > 0$

- Moreover $C_\theta(e,\theta) < 0$ and $C_{e\theta}(e,\theta) < 0$

Note that now the wage depends on the observable e: $w(e)$

So the utility is $U(w, e; \theta) = w(e) - c(e, \theta)$.

We assume $r(\theta) = 0$, so in a competitive market with no signaling all types are employed at a wage $E(\theta)$.

A competitive equilibrium with signalling is now a competitive equilibrium for each e.

So a $\Theta(e), w(e)$ such that:

$$w(e) = E(\theta; e \in \Theta(e))$$

and

$$\Theta(e) = \left\{\theta \text{ st. } e \in \arg\max_{e} U(w(e), e; \theta)\right\}$$

Ingredients: **indifference curves and wage function**

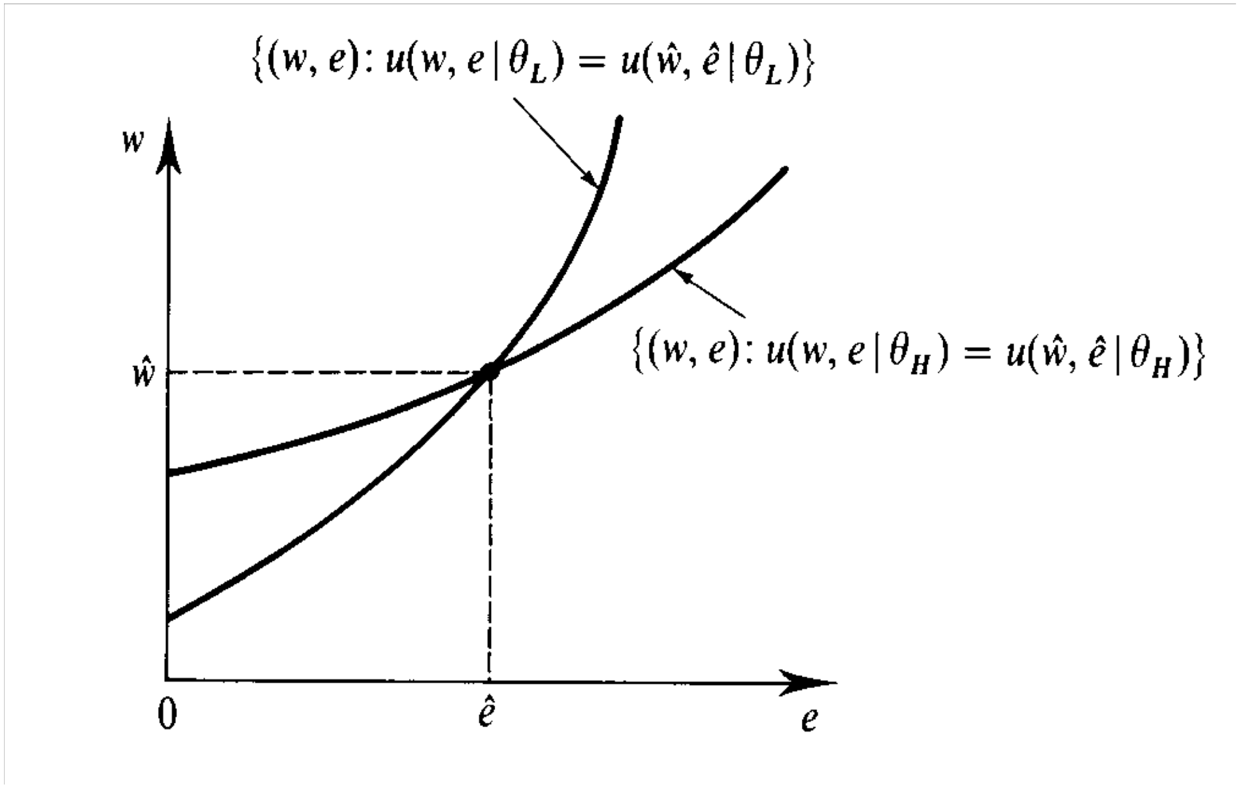
We can plot indifference curves in the w, e space.

Indifference curves for H and L types cross only once

The indifference curves of high types are less steep, since:

$$\frac{dw}{de} = C_e(e, \theta)$$

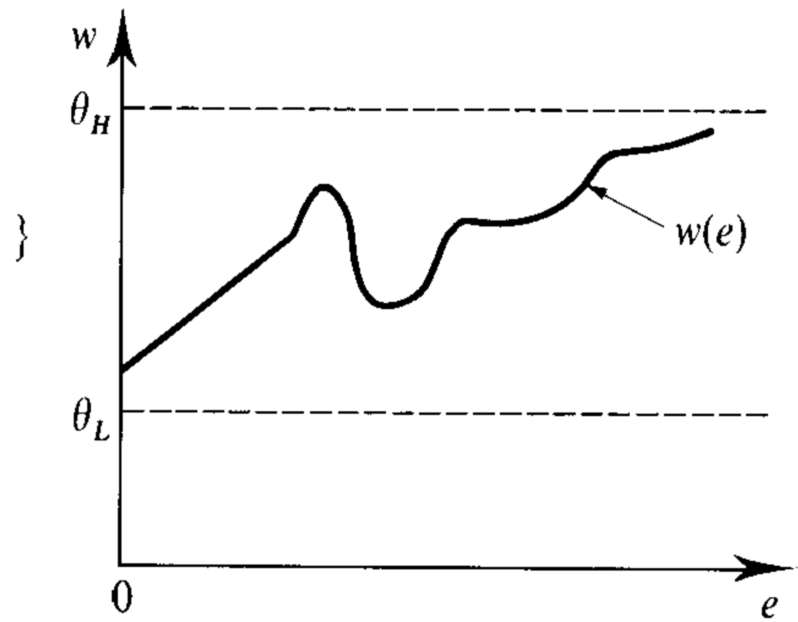
and $C_e(e, \theta_H) < C_e(e, \theta_L)$.


$\{(w, e): u(w, e \mid \theta_L) = u(\hat{w}, \hat{e} \mid \theta_L)\}$
w
$\hat{w}$
$\{(w, e): u(w, e \mid \theta_H) = u(\hat{w}, \hat{e} \mid \theta_H)\}$
0
$\hat{e}$
e

The wage function can be represented as:

$$w(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$$

where $\mu(e)$ is the posterior probability of a high type.


w
θ_H
$w(e)$
θ_L
0
e

Separating equilibria

In a separating equilibrium, the two types select different actions: $e^*(\theta_H) \neq e^*(\theta_L)$.

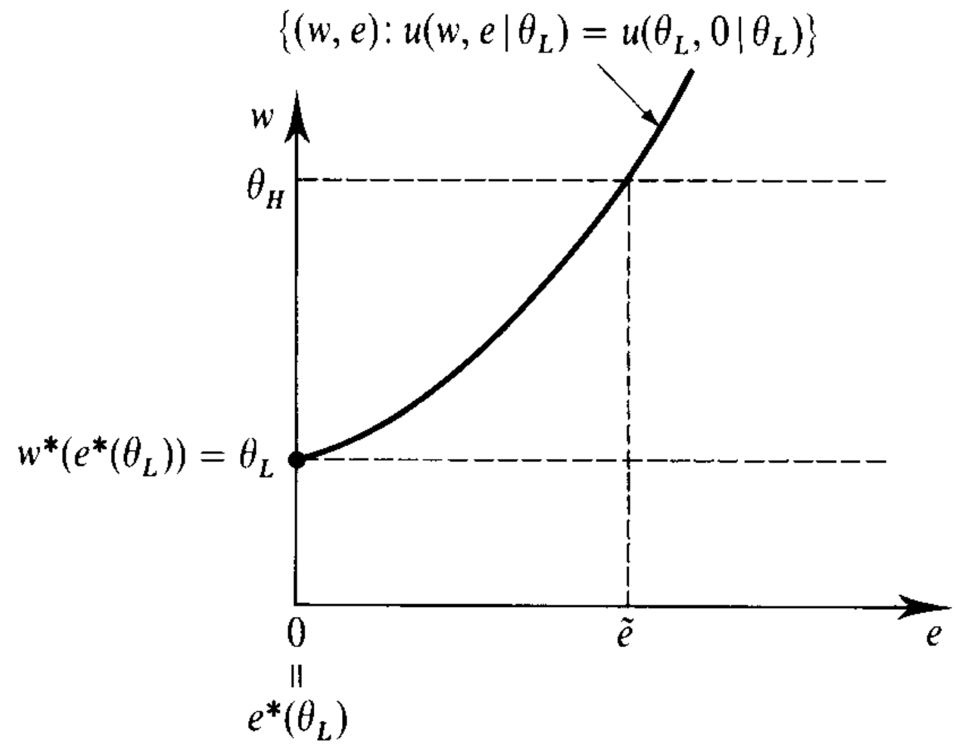
This immediately implies that:

Fact 1: In a separating eq.: $w^*(e^*(\theta_H)) \;\; = \theta_H$ and $w^*(e^*(\theta_L)) \;\; = \theta_L$.

Fact 2: we must have $e^*(\theta_L) = 0$.

Why? The low type has no benefit from $e^*(\theta_L)$, by choosing $e^*(\theta_L) = 0$, s/he gets the same wage and lower cost.

So the situation looks like this:



Starting from here, let us construct a separating equilibrium.

We need: $e^*(\theta_L) = 0$, and $e^*(\theta_H)$ such that:

$$U(w(e^*(\theta_H)), e^*(\theta_H); \theta_H) \geq U(w(e^*(\theta_L)), e^*(\theta_L); \theta_H)$$

$$U(w(e^*(\theta_L)), e^*(\theta_L); \theta_L) \geq U(w(e^*(\theta_H)), e^*(\theta_H); \theta_L)$$

Indeed, more generally:

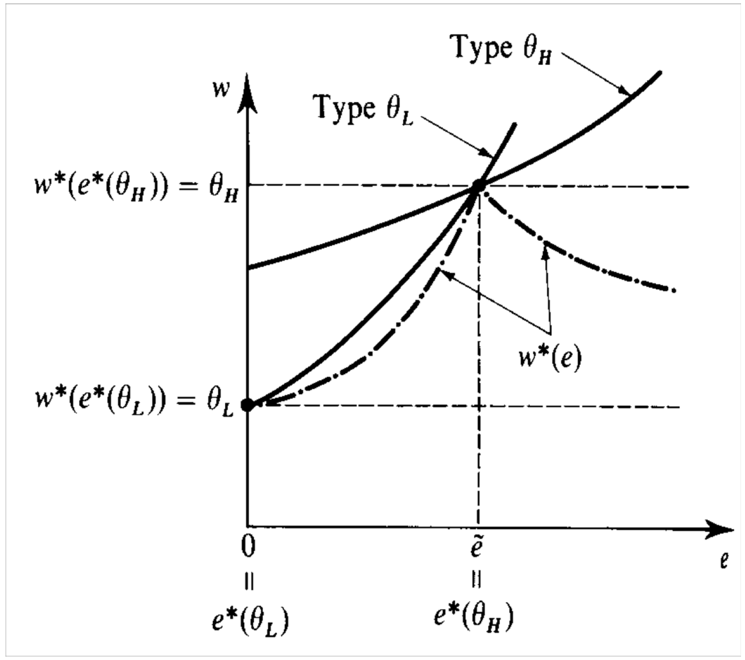
$$U(w(e^*(\theta_H)), e^*(\theta_H); \theta_H) \geq U(w(e)), e; \theta_H) \text{ IC(H)}$$

$$U(w(e^*(\theta_L)), e^*(\theta_L); \theta_L) \geq U(w(e), e; \theta_L) \text{ IC(L)}$$

for any e.

These constraints are called the incentive compatibility constraints.

Here is an example:



Note that here:

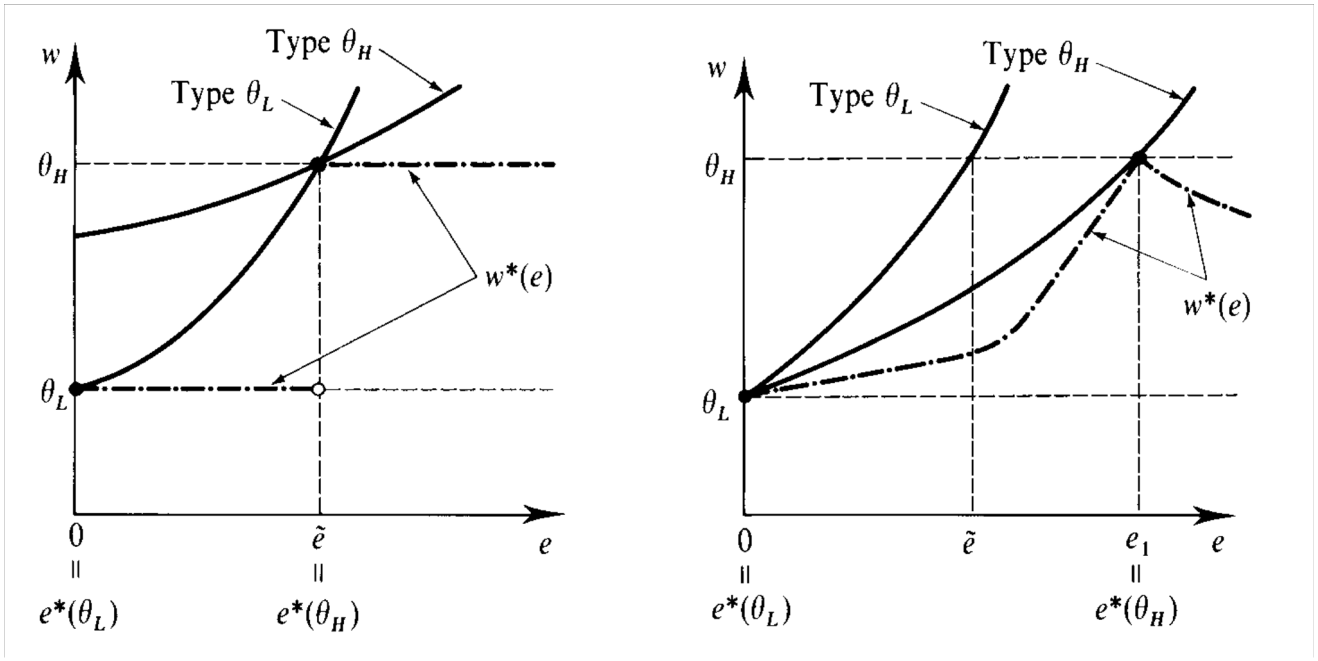
- $e^*(\theta_L) = 0$
- $U(w(e^*(\theta_L)), e^*(\theta_L); \theta_L) = U(w(e^*(\theta_H)), e^*(\theta_H); \theta_L)$
- $U(w(e^*(\theta_H)), e^*(\theta_H); \theta_H) \geq U(w(e^*(\theta_L)), e^*(\theta_L); \theta_H)$.

Note that we need to specify wages for all e: $w(e)$

This implies that we need to specify out of equilibrium beliefs for all e, even if they are never chosen since $w(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$, so:

$$\mu(e) = \frac{w(e) - \theta_L}{\theta_H - \theta_L}$$

We can have other equilibria:



The educational level of the high type cannot be lower that $\tilde{e}$, else we violate the IC constraint for the low type.

The educational level of the high type cannot be higher that e_1, else we violate the IC constraint for the high type (no education would be better than e).

The equilibria can be Pareto ranked:

- profits are always zero in a competitive equilibrium;
- effort is a net loss, so the lower e, the better.

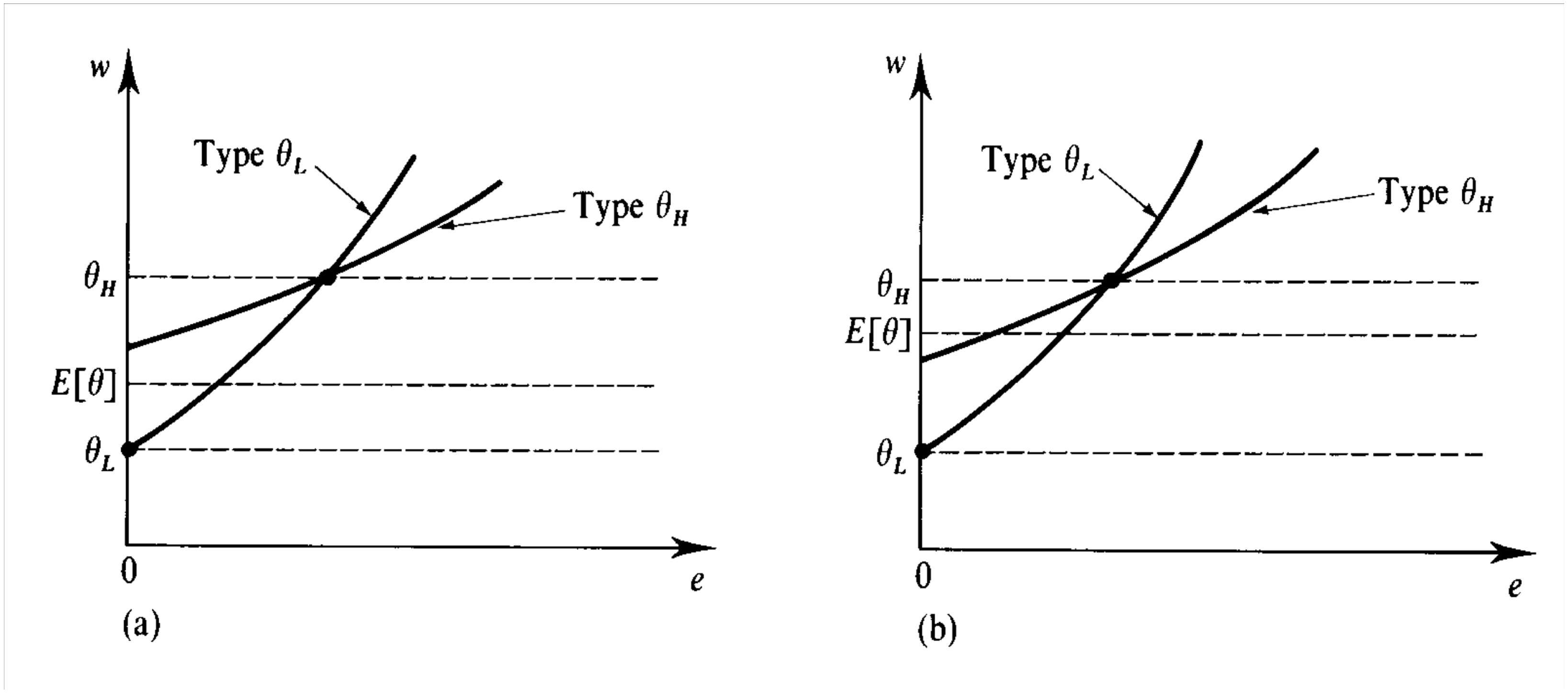
Are the players better off with signalling?

The low type is always worse off:

- Before signalling, s/he is employed at a wage $w = E\theta > 0$
- After signalling s/he is unemployed, or employed at wage $w = 0$.

The high type may be better off or worse off:

- Without signaling the utility is $U(E\theta, 0; \theta_H) = E\theta$.
- With signaling
  $U(w(e^*(\theta_H)), e^*(\theta_H); \theta_H) = w(e^*(\theta_H)) - e^*(\theta_H)$



In the equilibrium on the left:

$$U(w(e^*(\theta_H)), e^*(\theta_H); \theta_H) = \theta_H - C(e^*(\theta_H), \theta_H) > U(E\theta, 0; \theta_H)$$

In the eq. onthe right:

$$U(w(e^*(\theta_H)), e^*(\theta_H); \theta_L) < U(E\theta, 0; \theta_H)$$

Pooling equilibria

In pooling equilibria the two types choose the same action, so they are indistinguishable: $e^*(\theta_H) = e^*(\theta_L) = e^*$.

It follows that $w^*(e^*) = w^* = \lambda\theta_H + (1-\lambda)\theta_L = E\theta$

For this equilibrium we again need to define the wage for any e: $w(e)$.

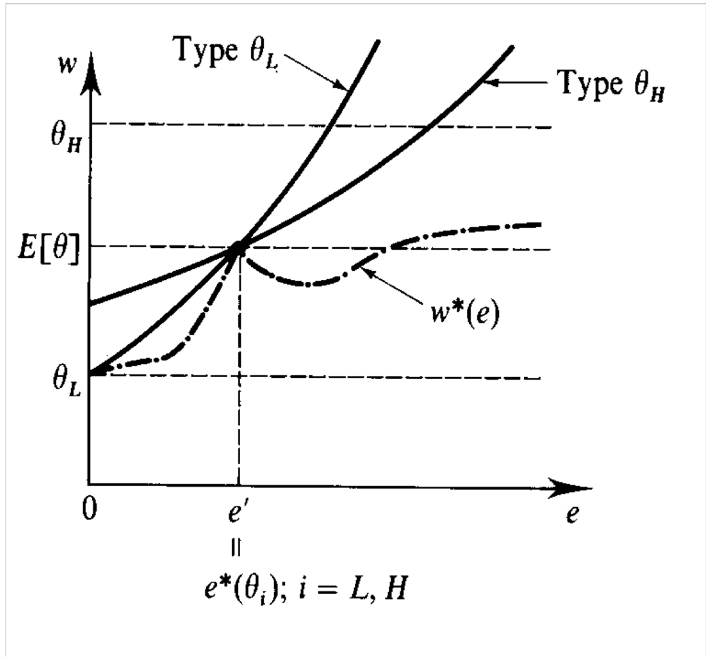
We need:

$$U(E\theta, e^*; \theta_H) \geq U(w(e), e; \theta_H) \text{ IC(H)}$$

$$U(E\theta, e^*; \theta_L) \geq U(w(e), e; \theta_L) \text{ IC(L)}$$

for any e.

Here is an example:



Multiple levels of efforts in a pooling equilibrium can be sustained by beliefs that "punish" lower levels of effort.

In this way we can sustain positive levels of effort also for low types.

The highest educational level is in correspondence to:

$$U(E\theta, e_1; \theta_L) = U(\theta_L, 0; \theta_L) = \theta_L$$

anything higher violates IC(L).

The lowest pooling equilibrium is $e = 0$.

Refinements

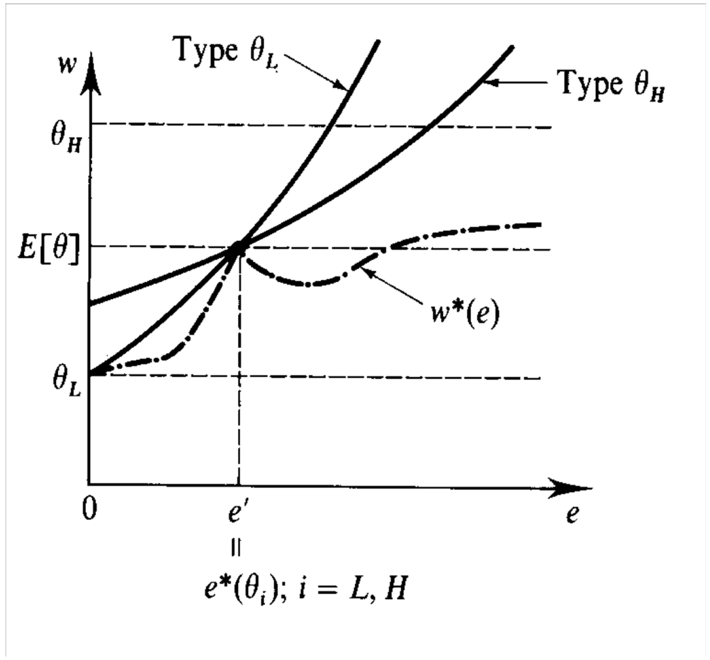
Consider a e' such that:

$E\theta - C(e^*, \theta_L) \geq \theta_H - C(e', \theta_L)$ and $E\theta - C(e^*, \theta_H) < \theta_H - C(e', \theta_H)$

where e^* is a pooling equilibrium.

- A low type is worse off if believed; a high type is better off;
- $\Rightarrow$A receiver would believe that the deviator is of high type.

Such a point e' always exists.



We may conclude that there are no pooling equilibria.

Consider now a separating equilibrium $e^L = 0, e^H$

In such an equilibrium $e^L = 0$ and $\theta_H - C(e^H, \theta_H) \geq \theta_L$.

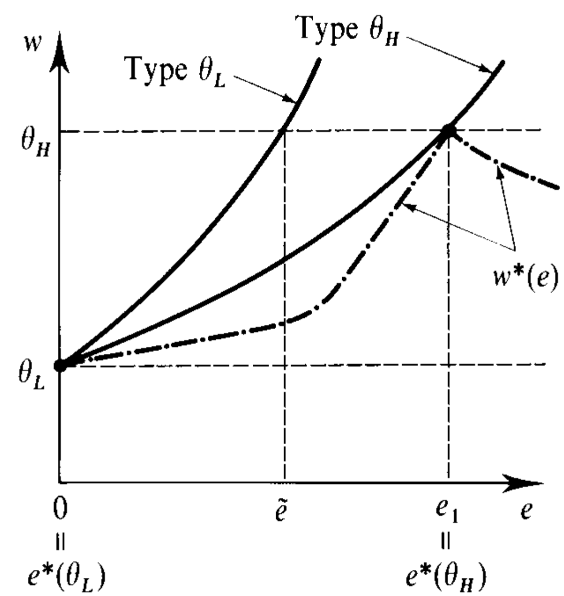
Assume $\theta_H - C(e^H, \theta_L) < \theta_L$.

If the receiver receives a deviation e' such that $e' < e^H$, but still $\theta_H - C(e', \theta_L) < \theta_L$:

- A high type benefits from the deviation if believed;
- A low type does not benefit if believed or not.
- Receiver concludes: deviation comes from the high type.

We conclude that there is only one separating equilibrium that survives in which:

$$e^L = 0 \text{ and } \theta_H - C(e^H, \theta_L) = \theta_L$$



Screening

In the previous analysis the informed agents pro-actively choose education to self select and credibly signal information.

When instead it is the uninformed agents who take measure to screen the informed agents, then we have screening.

Let us assume the same environment as before:

- there are two types $0 < \theta_L < \theta_H$, with $\Pr(\theta_H) = \lambda$.
- Now, $r(\theta) = 0$

We now assume that jobs can be assigned to different tasks:

- tasks are unproductive,
- but the cost effort: $C(t,\theta)$, with:

$$C(0,\theta) = 0, C_e(e,\theta) > 0, C_{ee}(e,\theta) > 0$$
$$C_\theta(e,\theta) < 0 \text{ and } C_{e\theta}(e,\theta) < 0$$

A contract is a pair $t, w(t)$

Let $\Theta(t) = \left\{\theta \text{ st. } t \in \arg\max_t u(w(t), t; \theta)\right\}$.

A family of contracts $(t, w(t))_t$ is a competitive equilibrium if:

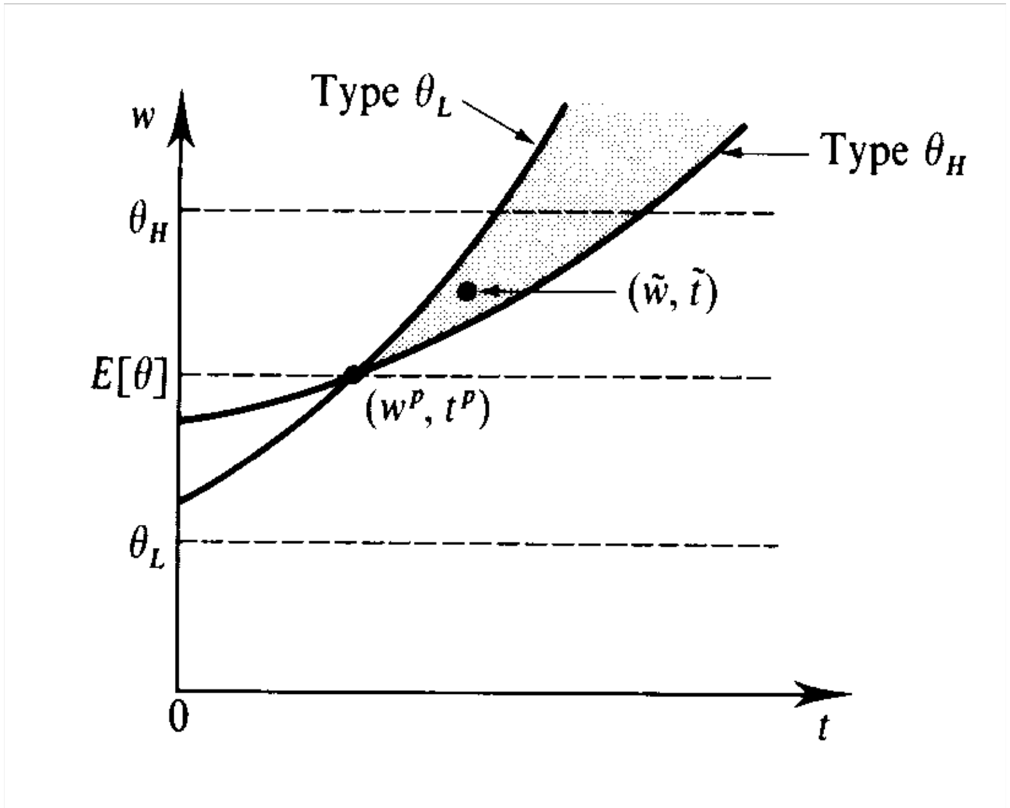
- $w(t) = E[\theta \text{ st. } \theta \in \Theta(t)]$,
- Profits are zero for all contracts $t, w(t)$.

In a competitive equilibrium with observable types $t, w(t,\theta) = 0, \theta$.

With unobservable types, we can have two cases:

- perfectly separating equilibria
- pooling equilibria
- (partially separating equilibria, but we will not really study them)

Our first result is that no pooling equilibrium can exist. See figure:

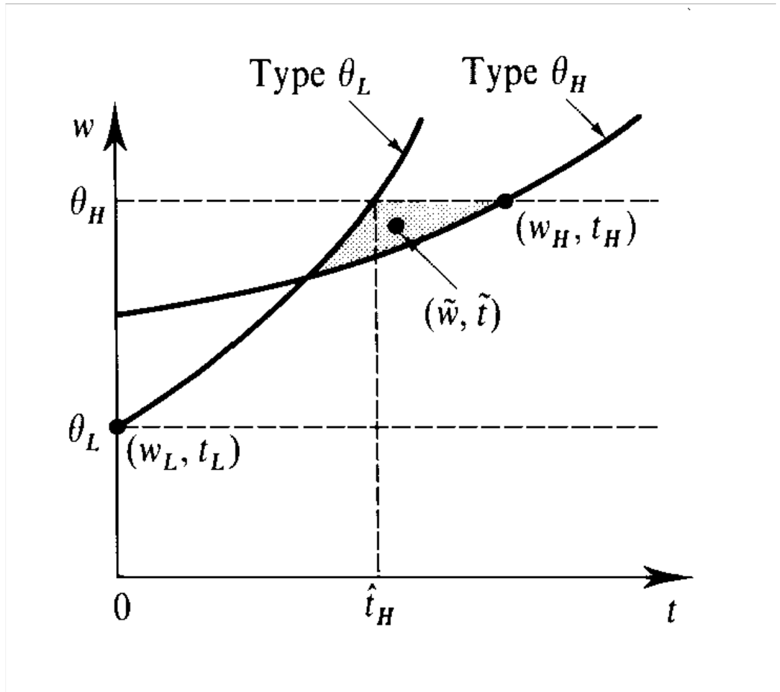


The second observation is that if w_L, t_L, w_H, t_H are the equilibrium contracts in a separating equilibrium, then $w_L, t_L = \theta_L, 0$ and $w_H, t_H = \theta_H, t_H^*$ such that:

$$\theta_H - C(t_H^*, \theta_L) = \theta_L - C(0, \theta_L)$$

So the low type is indifferent between $w_L, t_L = \theta_L, 0$ and the contract for the high type.

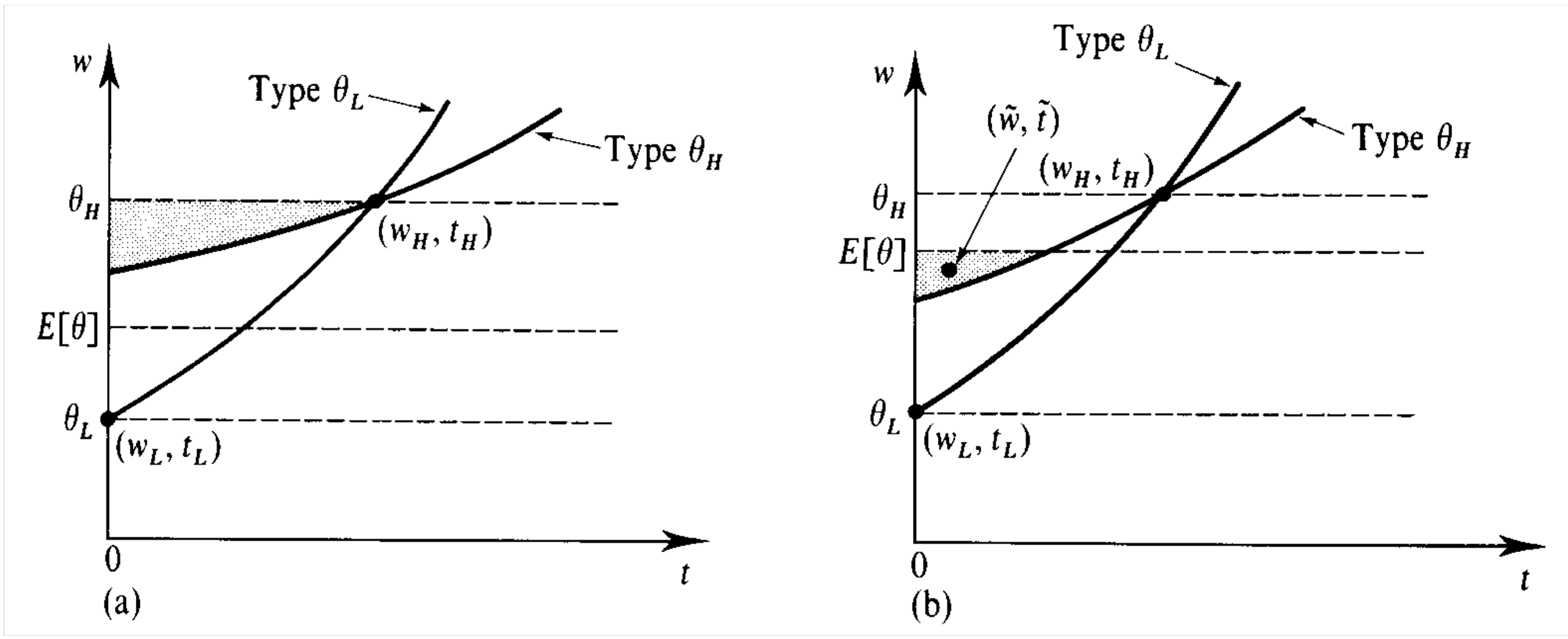
The intuition is as follows:



If an equilibrium does not exist, an equilibrium may exist in mixed strategies.

Existence

The problem is that an equilibrium in pure strategies may not exist.



Welfare

As for signaling, the low type is worse off with screening than without.

Now high types are always better off in a separating equilibrium.

The problem is that an equilibrium may not exist.

The market strikes back II: the principal agent model