Problem Set 1

Due: TA Discussion, 30 August 2024.

1 Exercises from class notes

Exercise 7 from "1. Real Sequences.pdf". Let S and T be nonempty and bounded subsets of $\mathbb{R}$. TFU: $\sup(S \cup T) = \max\{\sup S, \sup T\}$.

2 Additional Exercises

Exercise 1. Let A and B be nonempty subsets of $\mathbb{R}$. Define $A + B := \{a + b : a \in A \text{ and } b \in B\}$, and define $A - B$ similarly. Show the following:

(i) $\sup(A + B) = \sup(A) + \sup(B)$;

(ii) $\sup(A - B) = \sup(A) - \inf(B)$.

Exercise 2. Let A and B be nonempty sets, and let $f : A \times B \to \mathbb{R}$ be some real valued function.

(i) Show that

$$\sup_{a \in A} \inf_{b \in B} f(a, b) \leq \inf_{b \in B} \sup_{a \in A} f(a, b).$$

(ii) Give an example of a function $f : [0, 1]^2 \to \mathbb{R}$ for which the above inequality is strict.

Note: For a real valued function, f, on a nonempty set, S, $\sup_{x \in S} f(x) := \sup\{f(x) : x \in S\}$.