Welfare:

Say we have price and wealth change from (p^0, w^0) to (p', w')

$$CV = e(p', u') - e(p', u^0) \qquad \text{where} \quad u^0 = v(p^0, w^0)$$
$$EV = e(p^0, u') - e(p^0, u^0) \qquad\qquad\quad u' = v(p', w')$$

Special case: Only price of good 1 changes while other prices and wealth unchanged.

$$CV = e(p', u') - e(p', u^0)$$

$$= e(p^0, u^0) - e(p', u^0) \qquad \text{since } e(p', u') = e(p^0, u^0) = W.$$

$$= \int_{p_1'}^{p_1^0} \underbrace{h_1(t, \underset{=}{p_{-1}}, u^0)}_{\vee} \, dt.$$

$$EV = \int_{p_1'}^{p_1^0} h_1(t, \underset{\substack{= \\ \uparrow \\ (p_2, p_3 \cdots p_n)}}{p_{-1}}, u') \, dt$$

Remark: When price or wealth change makes you worse off, both CV and EV < 0.

Proposition: If x_1 is normal good, i.e. $\frac{\partial x_1}{\partial w} \geq 0$ then if only p_1 changes $EV \geq CV$.

Proof: Suppose WLOG $p_1' > p_1^0$.

It suffices to show that $h_1(t, p_{-1}, u') \geq h_1(t, p_{-1}, u^0)$ for all t.

Since we have $p_1' > p_1^0$, $u^0 \geq u'$

Since $h_1(p, u) = x_1(p, e(p, u))$

$$\frac{\partial h_1(p,u)}{\partial u} = \underbrace{\frac{\partial x_1(p, e(p,u))}{\partial w}}_{\geq 0 \;\; =0} \cdot \underbrace{\frac{\partial e(p,u)}{\partial u}}_{>0} \geq 0$$

$h_1(t, p_{-1}, u') \leq h_1(t, p_{-1}, u^0)$ for all t, since $u^0 \geq u'$.

$$\int_{p_1^0}^{p_1'} h_1(t, p_{-1}, u') \, dt \leq \int_{p_1^0}^{p_1'} h_1(t, p_{-1}, u^0) \, dt$$

$$EV = -\int_{p_1^0}^{p_1^1} h_1(t, p_{-1}, u^1)\, dt \gtrless -\int_{p_1^0}^{p_1^1} h_1(t, p_{-1}, u^0)\, dt = CV$$

Remark: If $\frac{\partial x_i}{\partial w} = 0$, then $CV = EV$ when p_i changes.

e.g. quasi-linear utility $u(x_1, x_2) = x_1 + f(x_2)$