ECON 6090-Microeconomic Theory. TA Section 11

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Exercise

MWG 6.C.15

The problem can be written as,

$$\max_{x_1,x_2} EU(x_1,x_2) = max_{x_1,x_2}\pi u(x_1+ax_2)+(1-\pi)u(x_1+bx_2)$$

Subject to

$$x_1+x_2=1$$

$$x_1,x_2\in[0,1]$$

If we substitute $x_1 = 1-x_2$, we get,

$$\max_{x_2\in[0,1]} EU(x_1,x_2) = max_{x_2\in[0,1]}\pi u(1-x_2+ax_2)+(1-\pi)u(1-x_2+bx_2)$$

(a) Since the decision maker is a risk averter, $u''(.)\le 0$.

$$\frac{\partial EU(x_2)}{\partial x_2} = \pi u'(1-x_2+ax_2)(a-1)+(1-\pi)u'(1-x_2+bx_2)(b-1)$$

$$\frac{\partial^2 EU(x_2)}{\partial x_2^2} = \pi u''(1-x_2+ax_2)(a-1)^2+(1-\pi)u''(1-x_2+bx_2)(b-1)^2\le 0$$

One necessary condition for the demand for the riskless asset to be strictly positive is,

$$\frac{\partial EU(x_2)}{\partial x_2}|_{x_2=1}< 0$$

Therefore,

$$\frac{\partial EU(x_2)}{\partial x_2}|_{x_2=1}= \pi u'(1-x_2+ax_2)(a-1)+(1-\pi)u'(1-x_2+bx_2)(b-1)|_{x_2=1}< 0$$

$$\implies \pi u'(a)(a-1)+(1-\pi)u'(b)(b-1)<0$$

Being a sufficient and necessary condition due to concavity.
Alternatively, a simple necessary condition is $\min\{a,b\}<1$.

(b) One necessary condition for the demand of the risky asset to be strictly positive is,

$$\frac{\partial EU(x_2)}{\partial x_2}|_{x_2=0}> 0$$

$$\implies \pi(a-1)+(1-\pi)(b-1)>0$$

(c) First order condition for the utility maximization,

$$\frac{\partial EU(x_2)}{\partial x_2} = \pi u'(1-x_2+ax_2)(a-1)+(1-\pi)u'(1-x_2+bx_2)(b-1)=0$$

(d) Notice that $x_1 = 1 - x_2 \implies dx_1 = -dx_2$, so,

$$\frac{dx_1}{da} \leq 0 \iff \frac{dx_2}{da} \geq 0$$

Then,

$$\frac{d}{da}\Bigg(\pi u'(1-x_2+ax_2)(a-1) + (1-\pi)u'(1-x_2+bx_2)(b-1)\Bigg) = 0$$

$$\pi(a-1)u''(1-x_2+ax_2)(-\frac{dx_2}{da} + x_2 + a\frac{dx_2}{da}) + \pi u'(1-x_2+ax_2) + (1-\pi)u''(1-x_2+bx_2)(b-1)^2\frac{dx_2}{da} = 0$$

$$\implies \frac{dx_2}{da} = -\frac{\pi(a-1)u''(1-x_2+ax_2)x_2 + \pi u'(1-x_2+ax_2)}{\pi(a-1)^2u''(1-x_2+ax_2) + (1-\pi)u''(1-x_2+bx_2)(b-1)^2}$$

A particular analysis of the signs of the ratio above taking into account that $u''(.) \leq 0$ and $a < 1$, gives,

$$\frac{dx_2}{da} \geq 0$$

(e) Since π is the probability of getting a payment of a on the risky asset, and we know $a < 1$, the larger pi gets, the more we will invest in the riskless asset instead, therefore $\frac{dx_1}{d\pi} > 0$.

(f) In this case we take derivative of π on FOC.

$$u'(1-x_2+ax_2)(a-1) + \pi(a-1)u''(1-x_2+ax_2)(-\frac{dx_2}{d\pi} + a\frac{dx_2}{d\pi}) - (b-1)u'(1-x_2+bx_2)$$

$$+(1-\pi)(b-1)u''(1-x_2+bx_2)(-\frac{dx_2}{d\pi} + b\frac{dx_2}{d\pi}) = 0$$

$$u'(1-x_2+ax_2)(a-1) - (b-1)u'(1-x_2+bx_2) + \pi(a-1)^2u''(1-x_2+ax_2)\frac{dx_2}{d\pi}$$

$$+(1-\pi)(b-1)^2u''(1-x_2+bx_2)\frac{dx_2}{d\pi} = 0$$

$$\Bigg[(1-\pi)(b-1)^2u''(1-x_2+bx_2) + \pi(a-1)^2u''(1-x_2+ax_2)\Bigg]\frac{dx_2}{d\pi} = (b-1)u'(1-x_2+bx_2) - u'(1-x_2+ax_2)(a-1)$$

$$\implies \frac{dx_2}{d\pi} = \frac{(b-1)u'(1-x_2+bx_2) - u'(1-x_2+ax_2)(a-1)}{(1-\pi)(b-1)^2u''(1-x_2+bx_2) + \pi(a-1)^2u''(1-x_2+ax_2)}$$

$$\implies \frac{dx_2}{d\pi} \leq 0$$

$$\implies \frac{dx_1}{d\pi} > 0$$