Department of Economics
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Final Exam
ECON6140: Spring 2023

Answer the following questions to the best of your ability. Points for each sub-question are given in parentheses.

To receive credit, you must show your work. An ideal answer will take a form similar to appendix material in a research paper. The logical argument should be made clearly and concisely. Moreover, each step should be introduced with enough words that the reader can understand its objective.

Section	Score
Q1	/35
Q2	/20
Q3	/20
Q4	/15
Total	/90

1. **Model with production externality.** Consider an economy populated by identical producer-consumer households with preferences for consumption given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t).$$

Households are endowed with an initial stock of capital, K_0. Household-level capital evolves according to the equation

$$K_{t+1} = (1-\delta)K_t + I_t \tag{1}$$

Household-level consumption is equal to output minus investment,

$$C_t = A_t K_t^{\alpha} - I_t \tag{2}$$

where $Y_t = A_t K_t^{\alpha}$ is the production function, and $0 < \alpha < 1$.

Finally, assume that total factor productivity, A_t, is taken as exogenous by each household, but depends on the aggregate level of current capital according to

$$log(A_t) = a_0 log(\bar{K}_t), \tag{3}$$

with $a_0 \geq 0$ and $a_0 + \alpha < 1$. In equilibrium, $K_t = \bar{K}_t$.

(a) In the language of the course, list separately the endogenous jump variables, the endogenous state variables, and the exogenous state variables in this model. Finally, make a list of all of the exogenous parameters of this economy. (5 points)

(b) Write the household's Lagrangian optimization problem and find the first order necessary conditions for optimality of the household. Denote the multipliers on constraints (1) and (2) with $\lambda_{1,t}$ and $\lambda_{2,t}$, respectively. (10 points)

(c) Write the social planner's Lagrangian optimization problem and find the first order necessary conditions for optimality. Denote the multipliers on constraints (1), (2), and (3) with $\theta_{1,t}, \theta_{2,t}$ and $\theta_{3,t}$ respectively. (5 points)

(d) Now write the Bellman equation that corresponds to the social planner's optimization problem in this economy and find the first order necessary conditions for optimality using the envelope theorem. Show that the conditions from (1.c) and (1.d) are equivalent. (10 points)

(e) Suppose, just for this part (e), that $\delta = 1$ and $a_0 > 0$. Using your results above, prove that the steady-state level of capital in the decentralized economy is less than the corresponding level in the social planner solution. Provide verbal intuition for your result. (5 points)

2. **Solution I.** Here we are going to take some steps towards solving the decentralized version of the model.

   (a) Combine your equations from part (1.b) — the household problem — into two equations in K_{t+1} and C_t (eliminating other variables.). (5 points)

   (b) Compute the steady-state values of K and C in terms of the model parameters. (5 points)

   (c) Log-linearize the equations you derived above from first principals. (i.e. replace C with $exp(c)$, etc and compute a first-order Taylor approximation.) You should log-linearize the equations around the steady-state, and you may treat the steady-state value of K and C as parameters, so you don't need to substitute in your answers from 2.b above (although doing so sometimes does give nicer expressions). (5 points)

   (d) Using the log-linearized system above, compute the F_x, F_y, F_{xp}, F_{yp} matrices that would be required for the log-linearization solution procedure we used in class. (5 points)

3. **Solution II.** Here we are going to take some steps towards solving the planner's version of the economy.

   (a) Using pseudo-code, describe an algorithm that solves for the approximate numerical value function you found in (1.d) over a finite grid of points `kgrid`. Below, I proved some initial steps. Your code does not need to compile, but you should pay special attention to indexing, so that a naive programmer could implement your algorithm. Also, be sure to test for convergence of your iterations. Do not include a "policy iteration" step in your algorithm. (20 points)

```
File: solve_vf.m

params = parameters; %Copies parameters into workspace
model_ss(params);    %Copies steady-state values for A, K, C into workspace

%Capital grid
kgrid = linspace(.8*kss, 1.2*kss,nk);

%Initial guess for value function
vf = 0*kgrid

%Largest number of iterations
maxiter = 10000
```

(continued on next page)

```
%main loop to update value function
tt = 0;     %iteration counter
crit = inf; %initial convergence criterion

while tt < maxiter && crit > 1e-9
       %Now you complete in bluebook using pseudo code

end
```

4. **Heterogenous agents.** Consider a version of our decentralized economy, except that now productivity also contains a household specific shock. That is,

$$log(A_t^i) = a_0 log(\bar{K}_t) + \epsilon_{it} \tag{4}$$

where ϵ_{it} is i.i.d. and normally distributed with variance σ and $\bar{K}_t \equiv \int K_t^i di$.

Otherwise, the economy is identical to the one described above with the addition of i subscripts. Consumers maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t^i).$$

Household-level capital evolves according to the equation

$$K_{t+1}^i = (1-\delta)K_t^i + I_t^i$$

Household-level consumption is equal to output minus investment,

$$C_t^i = A_t^i (K_t^i)^\alpha - I_t^i$$

where $Y_t^i = A_t^i (K_t^i)^\alpha$ is the production function, and $0 < a_0 + \alpha < 1$, while $a_0 > 0$ and $\alpha > 0$.

(a) Propose and describe an algorithm that would allow you to numerically solve for the steady level $\bar{K}_t$ as well as the steady-state cross sectional distribution of capital in the economy, $f(K^i)$. Think of your audience as a classmate who has a good understanding of the material from our course, but needs help solving this question as a problem set. Your description does not need to be as detailed as a "pseudo-code" but should include

   i. your approach to approximation for any policy functions you compute
   ii. your strategy for computing the expectations that influence in optimal choices
   iii. your strategy for solving for (approximate) optimal policy functions
   iv. your approach to aggregating individual-level behavior

   (15 points)