ECON 6090-Microeconomic Theory. TA Section 8

Omar Andujar

December 6, 2024

In Section notes

Uncertainty and Objective Expected Utility

Define $\succsim$ on $\mathcal{P}$ probability distribution

1. Suppose $\succsim$ on X. Then there will be limitations. It is not possible to incorporate risk preferences.
2. Incorporate $\succsim$ on X
   (a) $p_1 = (1, 0, 0) \succsim p_2 = (0, 1, 0) \implies x_1 \succsim x_2$

If $\succsim$ are rational and continuous, then there exists $V : \mathcal{P} \to \mathbb{R}$ such that $V(p)$ represents $\succsim$.

If $\succsim$ are rational, continuous and independent, then there exists $U : X \to \mathbb{R}$ such that $V(p) = \sum_x p(x)u(x)$. Here $u(x)$ is often called a Bernoulli utility function and $V(p)$ a Von Neumann-Morgenstern Objective Expected Utility.

Note that independence means: $\forall p, q \in \mathcal{P}\ \forall \alpha \in [0, 1]$, $p \succsim q \iff \alpha p + (1-\alpha)r \succsim q + (1-\alpha)r$.

Remark: If $V(.)$ and $U(.)$ both represent $\succsim$, then there exists a positive affine transformation between V and U. This also holds for their associated Bernoulli utility functions.

Exercises

2016 Prelim 2

We are given,

$$(\frac{1}{8}, \frac{3}{4}, \frac{1}{8}) \succsim (0, 1, 0)$$

$$(0, 1, 0) \succsim (\frac{1}{2}, 0, \frac{1}{2})$$

Assuming we have an objective utility representation,

$$\frac{1}{8}u(x_1) + \frac{3}{4}u(x_2) + \frac{1}{8}u(x_3) > u(x_2)$$

$$u(x_2) > \frac{1}{2}u(x_1) + \frac{1}{2}u(x_3)$$

Since $2u(x_2) > u(x_1) + U(x_3)$,

$$\frac{1}{8}(u(x_1) + u(x_3)) + \frac{3}{4}u(x_2) > u(x_2)$$

$$\implies \frac{1}{4}u(x_2) + \frac{3}{4}u(x_2) > u(x_2)$$

But this is a contradiction. Therefore, we cannot have an objective utility representation with these preferences.

2014 June Q

(a) The problem is,

$$EU(x) = \max_x pu(w - x + (1+r)x) + (1-p)u(w - x + (1+l)x)$$

To show that the individual will invest a positive amount of wealth $x > 0$ in the risky asset, it suffices to show that $\frac{\partial EU(x)}{\partial x}|_{x=0} > 0$. We observe that,

$$\frac{\partial EU(x)}{\partial x} = pu'(w - x + (1+r)x)(r) + (1-p)u'(w - x + (1+l)x)(l)$$

Then,

$$\frac{\partial EU(x)}{\partial x}|_{x=0} = u'(w)[pr + (1-p)l]$$

By assumption, $pr + (1-p)l > 0$ (actuarially favorable) and $u'(w) > 0$, so $\frac{\partial EU(x)}{\partial x}|_{x=0} > 0$.
Takeaway: A risk-averse agent always wants to invest a positive amount in actuarially favorable assets.

(b) When does $\frac{\partial x^*}{\partial w} > 0$?
Firstly, characterize x^*,

$$x^* = argmaxEU(x)$$

The first-order condition of the maximization problem in (a) gives,

$$\frac{\partial EU(x^*)}{\partial x} = pu'(w - x^* + (1+r)x^*)(r) + (1-p)u'(w - x^* + (1+l)x^*)(l) = 0$$

Now we take derivative with respect to w in the FOC,

$$pu''(w + rx^*)(r)(1 + r\frac{\partial x^*}{\partial w}) + (1-p)u''(w + lx^*)(l)(1 + l\frac{\partial x^*}{\partial w}) = 0$$

$$\implies \frac{\partial x^*}{\partial w} = -\frac{pu''(w + rx^*)r + (1-p)u''(w + lx^*)l}{pu''(w + rx^*)r^2 + (1-p)u''(w + lx^*)l^2}$$

Since $p > 0$, $u''(.) < 0$, we have that $pu''(w + rx^*)r^2 + (1-p)u''(w + lx^*)l^2 < 0$. So,

$$\frac{\partial x^*}{\partial w} \iff pu''(w + rx^*)r + (1-p)u''(w + lx^*)l > 0$$

$$\iff \frac{u''(w + rx^*)}{u''(w + lx^*)} < -\frac{(1-p)l}{pr}$$

$$\iff \frac{u''(w + rx^*)}{u''(w + lx^*)}\frac{u'(w + lx^*)}{u'(w + rx^*)} < -\frac{(1-p)l}{pr}\frac{u'(w + lx^*)}{u'(w + rx^*)}$$

$$\iff \frac{A(w + rx^*)}{A(w + lx^*)} < -\frac{(1-p)l}{pr}\frac{u'(w + lx^*)}{u'(w + rx^*)}$$

Where $A(.)$ is the coefficient of absolute risk aversion.
Since by the FOC $-\frac{(1-p)l}{pr}\frac{u'(w+lx^*)}{u'(w+rx^*)} = 1$,

$$\implies A(w + rx^*) < A(w + lx^*)$$

Where $r \geq 0$, $l \leq 0$ and $A(.)$ is decreasing.

2022 Prelim 2

(a) The problem is,

$$EU(x) = \max_\beta \alpha ln(w\beta p_A) + (1-\alpha)ln(w(1-\beta)p_B)$$

The first order condition gives,

$$\frac{\partial EU(x)}{\partial \beta} = \frac{\alpha}{\beta} - \frac{1-\alpha}{1-\beta} = 0$$

Since for the second order condition we have $\frac{\partial^2 EU(x)}{\partial \beta^2} \leq 0$,

$$\implies \beta^* = \alpha$$

(b) Increasing p_A does not affect the amount invested in project A, since the optimal amount β^* only depends on α.

(c) Since $ln(w^{\frac{1}{2}}) = \frac{1}{2}ln(w)$ is just a monotonic transformation of our original Bernoulli utility function, we stay with the same preferences as before.