ECON 6090: Problem Set 3

Adam Harris, Cornell University

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1. (**Production possibilities set**) Consider the production possibilities set

$$Y = \left\{ (q, -z) \in \mathbb{R}^2_+ \times \mathbb{R}^2_- : z_1^{\alpha} z_2^{\beta} \geq \frac{{q_1}^2 + {q_2}^2}{2} \right\}$$

where $\alpha, \beta > 0$.

   (a) Find the conditional input demand function $z(w_1, w_2, q_1, q_2)$.

   (b) What is the marginal rate of transformation between output 1 and output 2? That is, given w_1, w_2, q_1, q_2, what is the proportional decrease in q_1 required to marginally increase q_2 while holding cost constant?

2. (**Cost minimization**) Consider a single-output firm with technology that can transform inputs $z \in \mathbb{R}^3_+$ into output according to the production function

$$f(z) = {z_1}^{\frac{1}{2}} {z_2}^{\frac{1}{4}} {z_3}^{\frac{1}{8}}$$

   (a) This production function is homogeneous degree α. Find α. What does this imply about the firm's cost function? Is the firm's marginal cost of production increasing or decreasing in q?

   (b) Derive the conditional input demand function $z(w, q)$.

   (c) Derive an expression for the firm's marginal cost of production, i.e., the derivative of the cost function with respect to q.

3. (**Cost minimization with a continuum of inputs**) Consider a single-output firm which takes as input a continuum of inputs rather than a discrete set of inputs. We now denote the quantity input of commodity j as $z(j)$ (rather than z_j as we did in the discrete-inputs cases). The production function is

$$f(z) = \left[\int_0^1 a(j) z(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

where $a(j)$ is a continuous function integrable on $[0,1]$ that reflects the relative productivities of the various inputs.[1]

(a) Derive the conditional input demand function $z(j, w, q)$. The price for input j is given by $w(j)$, where w is a continuous function integrable on $[0,1]$.

(b) How is the conditional input demand for input j affected by $a(j)$, the productivity of input j?

(c) Now suppose that the firm has market power in input markets. If the firm uses $z(j)$ units of input j, the per-unit input price is $w(j, z(j)) = \frac{1}{2}z(j)$. Find the cost-minimizing choice of inputs to produce $q = 1$ units of output.

4. (**Profit maximization with a non-smooth production function**) Consider a single-output firm with technology that can transform inputs $z \in \mathbb{R}^N_+$ into output according to the production function

$$f(z) = 2\sqrt{\min\{z_1, 2z_2, 3z_3, \ldots, Nz_N\}}$$

(a) Derive the unconditional input demand function.

(b) Now suppose that the firm has market power in the output market. If the firm produces quantity q, the per-unit price is $P(q) = q^{-\epsilon}$ where $\epsilon \in (1, \infty)$. Derive the firm's choice of inputs $z_1, \ldots, z_N$.

5. (**Producer theory in action**) De Loecker, Eeckhout, and Unger (QJE, 2020) is an influential paper on measuring market power. The approach described in this paper takes the cost minimization problem as a starting point. Read the first 11 pages of this article (through the end of Section II.B) paying particular attention to Sections II.A and II.B.

(a) In going from equation (6) to (7), the authors assert that "The Lagrange multiplier λ is a direct measure of marginal cost." Give a justification for this assertion.

(b) The authors' starting point in Section II.B is the cost minimization problem. However, the output price (the key component of the markup) does not feature in the CMP (recall that the only arguments of the cost function and conditional input demand function are w and q). Given this, why can the authors claim that this starting point leads to some insight about markups? Wouldn't it be more natural to use the profit maximization problem as a starting point?

[1] While a continuum of inputs may not immediately seem empirically relevant, this assumption and the functional form imposed for f in this problem are commonly used in, for example, models of international trade.