ECON 6090: Lecture notes on market power

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1 Firms without market power

Thus far in this course, we have focused on the behavior of price-taking firms. The profit maximization problem (PMP) for such a firm is

$$\max_{z \in \mathbb{R}_+^{L-1}} pf(z) - w \cdot z \tag{PMP}$$

Note that both p and w are fixed and not functions of z or q. For each input $j = 1, \ldots, L-1$, the first order condition is

$$pf_j(z) = w_j \tag{FOC-z}$$

where $f_j(z) = \frac{\partial f(z)}{z_j}$ is the marginal product of input j. This first order indicates that the *marginal revenue product* of input j (that is, the additional revenue the firm gets from using an additional unit of input j) is equal to the price of input j.

Taking this first order condition for two inputs i and j and dividing one by the other gives

$$\frac{f_i(z)}{f_j(z)} = \frac{w_i}{w_j} \tag{MRTS}$$

That is, the ratio of the marginal products—called the *marginal rate of technical substitution*—is equal to the ratio of input prices. Note that we could also derive this relationship from the first order conditions of the CMP (as opposed to the PMP); in other words, this is a necessary condition for cost minimization.

Recall from lecture that we can rewrite the PMP as a two-stage problem in which the firm first chooses quantity q and then chooses the cost-minimizing z to produce that quantity. Since $C(w, q)$ is the value function of the CMP, the PMP can be written as

$$\max_q pq - C(w, q)$$

The first order condition is

$$p = \frac{\partial}{\partial q} C(w, q^*) \tag{FOC-q}$$

This shows that the firm without market power chooses quantity such that price is equal to marginal cost. That is, the firm earns zero profit on the marginal unit. This does not necessarily imply that the firm earns zero profit; if marginal cost $\frac{\partial}{\partial q} C(w, q)$ is increasing in q, then the firm's profit is

$$\int_0^{q^*} \left[p - \frac{\partial}{\partial q} C(w, q') \right] dq' > 0$$

because although $p - \frac{\partial}{\partial q} C(w, q^*) = 0$, $p - \frac{\partial}{\partial q} C(w, q') > 0$ for all $q' < q^*$. In other words, the firm earns positive profit on each *inframarginal* unit of output.

2 Firms with output market power

We now consider a firm with output market power. For such a firm, the output price depends on the quantity the firm produces. Thus, the PMP is

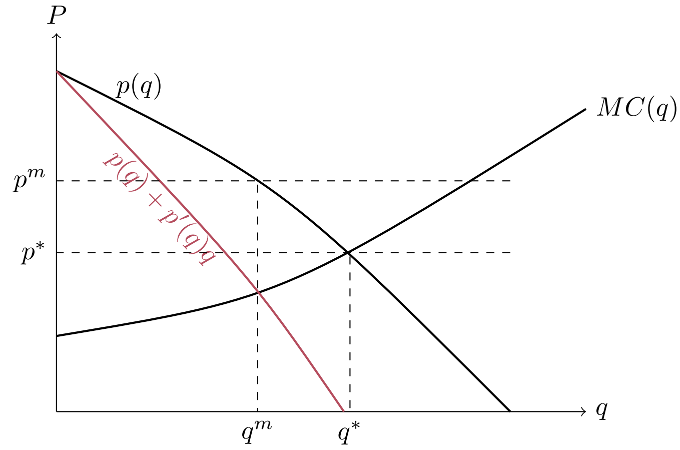
$$\max_{z \in \mathbb{R}_+^{L-1}} p(f(z)) f(z) - w \cdot z \tag{PMP-2}$$

where $p'(q) < 0$ for all $q \geq 0$. For each input $j = 1, \ldots, L-1$, the first order condition is

$$\left[\underbrace{p(f(z))}_{\text{marginal}} + \underbrace{p'(f(z)) f(z)}_{\text{inframarginal}} \right] f_j(z) = w_j \tag{FOC-z-2}$$

Comparing this to (FOC-z), the FOC for the firm without market power, we see that there is now an additional term. Rather than simply having the marginal revenue product $pf_j(z)$ equal to w_j, there is now an additional term $p'(f(z))$ on the left-hand side. The first term, as before, reflects the effect on revenue from the *marginal unit*; increasing z_j leads to an increase in output, and (as in the case without market

Figure 1: Quantity choice of firm with output market power



power) the firm earns an additional p for every additional unit of output. The second term reflects the effect on revenue from the *inframarginal units*; increasing z_j leads to an increase in output, which in turn leads to a decrease in p. This means that the firm earns slightly less revenue from each of the $f(z)$ units of output it was already producing.

Taking this first order condition for two inputs i and j and dividing one by the other gives, as before,

$$\frac{f_i(z)}{f_j(z)} = \frac{w_i}{w_j} \tag{MRTS-2}$$

The marginal rate of technical substitution still equals the ratio of input prices when the firm has market power. This makes sense since, as we showed in the lecture slides, (PMP-2), like (PMP), can be rewritten as a two-stage problem in which the firm first chooses quantity q and then chooses the cost-minimizing z to produce that quantity. Output market power may affect the firm's choice of q, but it will not lead the firm to produce that quantity in a non-cost-minimizing way. That is, the PMP can be written as

$$\max_q p(q)q - C(w, q)$$

The first order condition is

$$\underbrace{p(q)}_{\text{marginal}} + \underbrace{p'(q)q}_{\text{inframarginal}} = \frac{\partial}{\partial q}C(w, q) \tag{FOC-q-2}$$

This equation suggests how output market power distorts the firm's choice of q. Because $p'(q) < 0$ (that is, the demand curve is downward sloping), the inframarginal term, which captures the fact that increasing q decreases the revenue from each of the units already being produced, is negative. Rearranging this equation, we see that

$$p(q) = \frac{\partial}{\partial q}C(w, q) - p'(q)q > \frac{\partial}{\partial q}C(w, q) \tag{Margins-2}$$

This equation has two implications:

1. The firm with market power earns positive profit on *every* unit of output, including the marginal one.
2. If the firm has increasing marginal cost, then the firm with output market power chooses a quantity lower than the one that sets price equal to marginal cost.

That is, output market power tends to result in firms choosing lower quantities and higher prices. See Figure 1 for an illustration of these implications. In the figure, p^m, q^m represent the price and quantity of the firm with market power; p^*, q^* represent the price and quantity of a firm that chooses quantity such that price equals marginal cost.

3 Firms with input market power

We now consider a firm with input market power. For such a firm, the input price depends on the amount of that input the firm uses the firm produces.

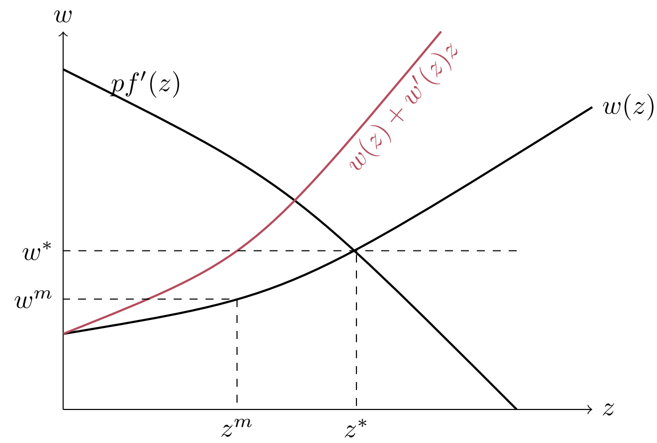
For simplicity, we consider the single-input case; that is $z \in \mathbb{R}_+$. For this case, the PMP is

$$\max_{z \in \mathbb{R}_+} pf(z) - w(z) \cdot z \tag{PMP-3}$$

where $w'(z) > 0$ for all $z \geq 0$. That is, the more of the input the firm uses, the greater the price it pays for each unit of the input. The first order condition is

$$pf'(z) = \underbrace{w(z)}_{\text{marginal}} + \underbrace{w'(z)z}_{\text{inframarginal}} \tag{FOC-z-3}$$

Figure 2: Input choice of firm with input market power



Comparing this to (FOC-z), the FOC for the firm without market power, we see that there is now an additional term. Rather than simply having the marginal revenue product $pf'(z)$ equal to w, $w'(z)z$ now also appears on the right-hand side. A marginal increase in z now has two effects on cost. The first term, as before, reflects the effect on cost from the *marginal unit* of input; increasing z means that (as in the case without market power) the firm must pay w for the additional unit of input. The second term reflects the effect on cost from the *inframarginal units* of input; increasing z leads to an increase in w. This means that the firm pays slightly more for each of the z units of input it was already using.

This equation suggests how input market power distorts the firm's choice of z. Because $w'(z) > 0$ (that is, the input supply curve is upward sloping), the inframarginal term, which captures the fact that increasing z increases the price of each of the units already being produced, is positive. Rearranging this equation, we see that

$$w(z) = pf'(z) - w'(z)z < pf'z(z) \tag{Margins-3}$$

This equation has two implications:

1. The firm with input market power pays less than its marginal revenue product for *every* unit of input, including the marginal one.

2. If the firm has a production function with decreasing marginal product (i.e., if $f''(z) < 0$), then the firm with input market power chooses to use less of input z than the one that sets input price equal to marginal revenue product.

That is, input market power tends to result in firms choosing lower z, which results in lower input prices. See Figure 2 for an illustration of these implications. In the figure, z^m, w^m represent the input quantity and input price of the firm with input market power; z^*, w^* represent the input quantity and input price of a firm that chooses z such that input price equals marginal revenue product.