Prospect Theory (Kahneman & Tversky, 1979)

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Expected Utility Theory

Let w be a person's *wealth*.

Let $\mathbf{x} \equiv (x_1, p_1; ...; x_n, p_n)$ be a risky prospect.

- $\mathbf{x}$ yields *income* x_i with probability p_i.
- $\sum_{i=1}^{n} p_i = 1$.

EU theory says evaluate prospect $\mathbf{x}$ according to utility function

$$U(\mathbf{x}; w) = p_1 u(w + x_1) + ... + p_n u(w + x_n).$$

That is: Choose prospect $\mathbf{x}$ over prospect $\mathbf{y}$ if

$$U(\mathbf{x}; w) > U(\mathbf{y}; w).$$

Expected Utility Theory: Some Features

- u is a cardinal utility function—unique up to a positive affine transformation.

- Linear in the probabilities.

  - Derives from the **independence axiom**:

    If $\mathbf{x} \succeq \mathbf{y}$, then for any prospect $\mathbf{z}$ and $\alpha \in (0, 1)$,

$$\alpha\mathbf{x} + (1 - \alpha)\,\mathbf{z} \succeq \alpha\mathbf{y} + (1 - \alpha)\,\mathbf{z}.$$

- Subjective vs. objective probabilities

Expected Utility Theory: Some Features

- EU provides an appealing explanation for risk aversion.

Definition: A person is *globally risk-averse* if, for any lottery $\mathbf{x}$, she prefers a certain payment equal to $E\mathbf{x}$ over the lottery $\mathbf{x}$ itself; and she is *locally risk-averse over range* $[x', x'']$ if, for any lottery $\mathbf{x}$ with support a subset of $[x', x'']$, she prefers a certain payment equal to $E\mathbf{x}$ over the lottery $\mathbf{x}$ itself;

Result: Under EU theory, a person is globally risk-averse if and only if $u(\cdot)$ is globally concave, and she is locally risk-averse over range $[x', x'']$ if and only if $u(\cdot)$ is concave over range $[x', x'']$.

Note: There exist analogous definitions and results for being risk-seeking and risk-neutral.

Expected Utility Theory: Some Features

- Integration: *EU* operates on final wealth states (or final consumption bundles).

  Consider a 50-50 bet to win \$1000 vs. lose \$950.

  - Proper use of *EU* is

$$U(\mathbf{x}; w) = \frac{1}{2} u(w + 1000) + \frac{1}{2} u(w - 950)$$

  - Do NOT use

$$U(\mathbf{x}; w) = \frac{1}{2} u(1000) + \frac{1}{2} u(-950)$$

Prospect Theory: Evidence

A few details on the evidence:

- Asked students and faculty to respond to **hypothetical** choice problems, originally in Israel, later replicated at Stockholm and Michigan (note: median net monthly income in Israel $\approx$ 3000).
- Series of binary choices between two prospects; no more than a dozen problems per questionnaire; usual techniques of varying order of questions and positions of choices.
- Their notation eliminates \$0 outcomes — e.g., "(4000,.8)" means 4000 with probability 0.8, 0 with probability 0.2.

Prospect Theory: Evidence

	Option (A)	vs.	Option (B)
Problem 1	2500 with prob .33		2400 with prob 1
$[N = 72]$	2400 with prob .66		
	0 with prob .01		

^^^

	Option (C)	vs.	Option (D)
Problem 2	2500 with prob .33		2400 with prob .34
$[N = 72]$	0 with prob .67		0 with prob .66

Prospect Theory: Evidence

(B) $\succ$ (A):

$$u(w+2400) > .66u\left(w+2400\right) + .33u\left(w+2500\right) + .01u\left(w\right)$$

or

$$.34u(w+2400) > .33u\left(w+2500\right) + .01u\left(w\right)$$

(C) $\succ$ (D):

$$.33u\left(w+2500\right) + .67u\left(w\right) > .34u\left(w+2400\right) + .66u\left(w\right)$$

or

$$.33u\left(w+2500\right) + .01u\left(w\right) > .34u\left(w+2400\right)$$

Prospect Theory: Evidence

	Option (A)	Option (B)
Problem 7	6000 with prob .45	3000 with prob .90
$[N = 66]$	0 with prob .55	0 with prob .10

^^

	Option (C)	Option (D)
Problem 8	6000 with prob .001	3000 with prob .002
$[N = 66]$	0 with prob .999	0 with prob .998

Prospect Theory: Evidence — Subproportionality

From these and similar examples, Kahneman & Tversky conclude there is "subproportionality":

- If $(y, pq) \sim (x, p)$ then $(y, pqr) \succ (x, pr)$ [where $y > x$ and $p, q, r \in (0, 1)$].

Prospect Theory: Evidence

Problem 7: $[N = 66]$	$(6000, .45)$ $[14\%]$	$\prec$	$(3000, .90)$ $[86\%]^*$
Problem 8: $[N = 66]$	$(6000, .001)$ $[73\%]^*$	$\succ$	$(3000, .002)$ $[27\%]$
Problem 7′: $[N = 66]$	$(-6000, .45)$		$(-3000, .90)$
Problem 8′: $[N = 66]$	$(-6000, .001)$		$(-3000, .002)$

Prospect Theory: Evidence — Reflection Effect

From these and similar examples, Kahneman & Tversky conclude that preferences exhibit a "reflection effect":

- Preferences over losses are the opposite of preferences over equivalent gains.

- Another feature: "four-fold pattern of risk preferences"
  - For intermediate probabilities, risk-averse behavior over gains and risk-loving behavior over losses.
  - For small probabilities, risk-loving behavior over gains and risk-averse behavior over losses.

Prospect Theory: Evidence

Problem 10: Consider the following two-stage game. In the first stage, there is a probability of .75 to end the game without winning anything, and a probability of .25 to move into the second stage. If you reach the second stage you have a choice between

$$(4000, .80) \qquad \text{and} \qquad (3000, 1).$$

Your choice must be made before the game starts, i.e., before the outcome of the first stage is known.

Note: we can collapse this to

"Problem 10": $(4000, .2)$ $(3000, .25)$
$[N = 141]$

Prospect Theory: Evidence

"Problem 10": $[N = 141]$	$(4000, .2)$		$(3000, .25)$

^^

Problem 3: $[N = 95]$	$(4000, .8)$ $[20\%]$	$\prec$	$(3000, 1)$ $[80\%]^*$
Problem 4: $[N = 95]$	$(4000, .2)$ $[65\%]^*$	$\succ$	$(3000, .25)$ $[35\%]$

Problem 11: You get 1000 for sure. In addition, choose between $[N = 70]$

$$(1000, .5) \qquad \text{vs.} \qquad (500, 1)$$

Problem 12: You get 2000 for sure. In addition, choose between $[N = 68]$

$$(-1000, .5) \qquad \text{vs.} \qquad (-500, 1)$$

Prospect Theory: Evidence — Isolation Effect

From these and similar examples, Kahneman & Tversky conclude that people exhibit an "isolation effect":

- People ignore seemingly extraneous parts of the problem — in particular, they tend to disregard shared components.

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Brief aside: There is now a large literature on "framing effects" — two ways of presenting the **exact same problem** elicit different choices.

The isolation effect is a natural interpretation of some framing effects — because for some ways of framing a problem, certain information can seem extraneous.

Prospect Theory (an alternative to EU Theory)

A theory for simple prospects with at most two non-zero outcomes.

- Note: A prospect can be written as $(x, p; y, q)$ with $p + q \leq 1$.
- Note: $p + q < 1$ implies prospect yields 0 with probability $1 - p - q$.

Two Phases of Choice Process:

- Editing
- Evaluation

Prospect Theory: Editing Stage

Editing Stage: organize & reformulate the problem

What's going on? Taking an "objective" prospect $(\hat{x}_1, \hat{p}_1; \ldots; \hat{x}_n, \hat{p}_n)$ and transforming it into an object for evaluation $(x_1, p_1; \ldots; x_m, p_m)$.

- Coding: code outcomes as gains & losses relative to reference point.
- Combination: e.g., $(100, .5; 100, .5)$ replaced with $(100, 1)$.
- Segregation: e.g., $(100, .5; 200, .5)$ replaced with 100 for sure plus $(0, .5; 100, .5)$.
- Cancellation: discard shared components.
- Simplification: rounding off probabilities.
- Eliminating dominated alternatives.

Prospect Theory: Evaluation Stage

A person evaluates a prospect $(x, p; y, q)$ according to the functional

$$V(x, p; y, q) \quad = \quad \pi(p)\, v(x) \quad + \quad \pi(q)\, v(y).$$

Reminder: EU theory says use

$$U(x, p; y, q) = pu(w + x) + qu(w + y) + (1 - p - q)\, u(w)$$

What's new?

- $\pi(\cdot)$ is the probability-weighting function.
- $v(\cdot)$ is the value function.

Prospect Theory: Value Function

Three key features of the value function $v(\cdot)$:

- The carriers of value are changes in wealth ($v(0) = 0$).
- *Diminishing sensitivity* to the magnitude of changes ($v''(x) < 0$ for $x > 0$, $v''(x) > 0$ for $x < 0$).
- *Loss aversion*: losses loom larger than gains.

Diminishing Sensitivity

- *Diminishing sensitivity* to the magnitude of changes $(v''(x) < 0$ for $x > 0$, $v''(x) > 0$ for $x < 0)$.

$$\begin{array}{llcl}
\text{Problem 13:} & (6000, .25) & \prec & (4000, .25; 2000, .25) \\
[N = 68] & [18\%] & & [82\%]^{*} \\
\\
\text{Problem 14:} & (-6000, .25) & \succ & (-4000, .25; -2000, .25) \\
[N = 64] & [70\%]^{*} & & [30\%]
\end{array}$$

Loss Aversion

- *Loss aversion*: losses loom larger than gains.

Based on introspection, they conclude:

Example: $(100, .5; -100, .5) \succ (1000, .5; -1000, .5)$

More generally: $(y, .5; -y, .5) \succ (x, .5; -x, .5)$

for any $x > y \geq 0$.

Prospect Theory: Probability-Weighting Function

Some key features of the probability-weighting function $\pi(\cdot)$:

- Natural assumptions: $\pi(0) = 0$, $\pi(1) = 1$, and π is increasing.
- For small p, $\pi(p) > p$.
- Subcertainty: $\pi(p) + \pi(1-p) < 1$.
- Subproportionality: $\pi(pq) / \pi(p) \leq \pi(pqr) / \pi(pr)$ for $p, q, r \in (0, 1)$.
- Discontinuity at endpoints.

Four Themes that Emerged from Prospect Theory

1. Non-linear decision weights.
2. Reference dependence & loss aversion.
3. Framing effects & mental accounting.
4. Experienced utility.

Reference Dependence and Loss Aversion

- Two common functional forms for the value function:

<u>Tversky & Kahneman (1992)</u>

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x \leq 0 \end{cases}$$

where $\alpha, \beta \in (0, 1]$ and $\lambda \geq 1$

<u>Two-part linear</u>

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x \leq 0 \end{cases}$$

where $\lambda \geq 1$

- A more general overall utility function:

$$U(x|r) \equiv u(x) + v(x - r)$$

  - x is final consumption, r is the reference point
  - $u(x)$ is intrinsic utility from consumption ("standard economic utility")
  - $v(x - r)$ is gain-loss utility