# Econ 6190 Problem Set 7

Fall 2024

1. Let $\{X_1 \dots X_n\}$ be a sequence of i.i.d random variables with mean $\mu$ and variance $\sigma^2$. Let $\hat{\mu} = \frac{1}{n}\sum_{i=1}^n X_i$, and $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^n (X_i - \hat{\mu})^2$.

   (a) Suppose $\mathbb{E}X_i^2 < \infty, i = 1, \dots n$. Show $\hat{\sigma}^2 \xrightarrow{p} \sigma^2$ as $n \to \infty$.

   (b) Imposing additional assumptions if necessary, find the asymptotic distribution of

   $$\sqrt{n}(\hat{\sigma}^2 - \sigma^2)$$

   by using delta method. Carefully state your results.

2. [Hansen 8.10] Let $X \sim U[0, b]$ and $M_n = \max_{i \le n} X_i$, where $\{X_i, i = 1 \dots n\}$ is a random sample from $X$. Derive the asymptotic distribution using the following the steps.

   (a) Calculate the distribution $F(x)$ of $U[0, b]$.

   (b) Show

   $$Z_n = n(M_n - b) = n\left(\max_{1\le i\le n} X_i - b\right) = \max_{1\le i\le n} n\left(X_i - b\right).$$

   (c) Show that the cdf of $Z_n$ is

   $$G_n(x) = P\{Z_n \le x\} = (F(b + \frac{x}{n}))^n.$$

   (d) Derive the limit of $G_n(x)$ as $n \to \infty$ for $x < 0$. [Hint: Use $\lim_{n\to\infty}(1 + \frac{x}{n})^n = e^x$]

   (e) Derive the limit of $G_n(x)$ as $n \to \infty$ for $x \ge 0$.

   (f) Find the asymptotic distribution of $Z_n$ as $n \to \infty$.