ECON 6090-Microeconomic Theory. TA Section 9

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In Section notes

Savage's Subjective Expected Utility

(.) X: set of outcomes

(.) S: set of states

(.) F: set of acts $\{f | f : S \to X\}$

(.) $\mathcal{P}$: Distribution over states (prior)

(.) $\succsim$: preference relation over F

(.) $u : X \to \mathbb{R}$. Utility function.

(.) $A = 2^S$. Set of all possible subsets of S.

Example:

$$S = \{1, 2, 3\}$$

$$A = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \emptyset\}$$

Where $1 \in S$ is a sample, and $\{1, 2\} \in A$ is an event.

Some definitions

1. $\forall h, f \in F$,

$$f|_A h(s) = \begin{cases} f(s) & s \in A \\ h(s) & s \notin A \end{cases}$$

2. $\forall x, y \in X$,

$$xAy = \begin{cases} x & s \in A \\ y & s \notin A \end{cases}$$

3. $\forall f, g \in F$, $f \succsim_A g$, if for some k, $f|_A k \succsim_A g|_A k$.

4. Event A is null if $\forall f, g \in F$, $f \succsim_A g$.

5. Sets are ordered $A \succsim B$ if and only if there exist an outcome $x \succ y$ such that $xAy \succsim xBy$.

Savage Axioms:

P1 The preference relation $\succsim$ on F is rational (complete and transitive).

P2 If $f|_A h \succ g|_A h$, then $f|_A k \succ g|_A k$ $\forall k \in F$.

(.) Preferences on acts only depend on where they differ. Example:

$$S = \{sunny(w_1), rainy(w_2)\}$$

$$X = \{hiking, sleeping, working\}$$

$$f = \begin{cases} hiking & w_1 \\ sleeping & w_2 \end{cases}$$

$$g = \begin{cases} working & w_1 \\ hiking & w_2 \end{cases}$$

$$q = \begin{cases} sleeping & w_1 \\ sleeping & w_2 \end{cases}$$

If $A = \{w_1\}$, $f|_A g \succ g|_A g \implies f|_A q \succ g|_A q$

P3 $\forall x, y \in X$, A non-null, $x \succsim_A y \iff x \succsim y$

P4 For outcomes $x \succ y$, $x' \succ y'$ and sets A,B:

$$xAy \succsim xBy \iff x'Ay' \succsim x'By'$$

Note that $xAy \succsim xBy \implies A \succsim B$.

P5 There exist outcomes $x \succ y$.

P6 (Small-event continuity) If $f \succsim g$ then for any consequence x there is a partition of S such that on each S_i, $f|_{S_i}h \succsim g$ and $f \succsim g|_{S_i}h$.

P7 If f and g are acts and A is an event such that $f(s) \succsim_A g$ for every $s \in A$, then $f \succsim_A g$; and if $f \succsim_A g(s)$ for every $s \in A$, then $f \succsim_B g$.

If $\succsim$ satisfies axioms P1-P5, we get the theorem that establishes the existence of a SEU,

$$f \succsim g \iff \int u(f(s))dp \geq \int u(g(s))dp$$

Exercises

Subjective Expected Utility

2014 Final

(a) The individual's decision problem is,

$$SEU(x) = \max_{x \in \mathbb{R}} \pi(S)u(w - px + Rx) + (1 - \pi(S))u(w - px)$$

Notice that SEU(X) is concave in x. Since ris averse $\iff$ $u(.)$ is concave. That also means that $E(U(x)) \leq U(E(X))$ (Jensen's Inequality).

(b) From the problem we can infer that $x = 0$ is optimal.

$$\implies \frac{\partial SEU(x)}{\partial x}|_{x=0} = 0$$

$$u'(w - px + Rx)\pi(S)(R - p) - u'(w - px)(1 - \pi(S))p|_{x=0} = 0$$

$$\implies \frac{u'(w)}{u'(w)} = \frac{(1 - \pi(S))p}{\pi(S)(R - p)}$$

$$\implies \pi(S) = \frac{p}{R}$$

(c) Assuming that "going short" is prohibited. That is, $x \geq 0$, a new individual that chooses 0 means,

$$\frac{\partial SEU(x)}{\partial x}|_{x=0} \leq 0$$

$$\implies \pi(S) \leq \frac{p}{R}$$

Otherwise, if $\frac{\partial SEU(x)}{\partial x}|_{x=0} > 0$, we can choose $x > 0$ and increase our subjective expected utility. Meaning that $x = 0$ is not the maximizer.