ECON 6090-Microeconomic Theory. TA Section 7

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In Section notes

Profit Maximization without market power

$$\pi(p,w) = \max_z pf(z) - w \cdot z$$

First order condition gives,

$$\text{(MRS)}\ \frac{f_i(z)}{f_j(z)} = \frac{w_i}{w_j}$$

Also,

$$\pi(p,w) = \max_z p \cdot f(z) - w \cdot z$$
$$= \max_{q,z} p \cdot q - w \cdot z \text{ s.t. } q = f(z)$$
$$= \max_q p \cdot q - \min_z w \cdot z \text{ s.t. } q = f(z)$$

Where CMP: $\min_z w \cdot z$ s.t. $q = f(z)$. So the profit maximization problem can be defined in two steps:

1. Find the cheapest way to produce q
2. Choose optimal q to maximize profit

Profit Maximization with market power
Your output affects the price, $p'(q) \leq 0$.

$$\max_q p(q)q - c(w,q)$$

First order condition gives,

$$\text{MR } = p(q) + p'(q)q = \frac{d}{dq}c(w,q) = MC$$

In optimal, $p^* > MC$
We want to choose less q since $p(q)$ is decreasing in q.

Profit Maximization with input market power

$$\max_z p \cdot f(z) - w(z) \cdot z$$

First order condition gives,

$$pf'(z) = w(z) + w'(z) \cdot z$$

Since $w'(z) \geq 0$

$$pf'(z) > w(z)$$

(Notice that $pf'(z) = w(z)$ in competitive markets) In this scenario, choose less z and produce less.

Exercises

Output market power

(a) First we find the demand function.
For a fixed p, v consumer will buy if and only if its utility from buying is positive. This means that,

$$\text{Consumer will buy} \iff \theta v - p \geq 0$$

Since we need $\theta \geq \frac{p}{v}$ and we know $\theta \in [0,1]$, our demand function becomes $D(p,v) = 1 - \frac{p}{v}$.
Our maximization problem becomes,

$$\pi(p,w) = \max_p p \cdot D(p) - cv^2 D(p)$$

$$= \max_p (p - cv^2)(1 - \frac{p}{v})$$

And our first order condition is,

$$1 - \frac{2p}{v} + cv = 0$$

Since $\frac{\partial \pi^2(p)}{\partial p} < 0$, the FOC give us the optimal value for p. That is,

$$p^* = \frac{1}{2}(v + cv^2)$$

Notice that if $p^* \geq v$ no one will buy and $\pi^* = 0$. On the other hand, if $p^* < v$, $\pi^* = \frac{1}{4}v(1+cv)^2 - cv^2$

(b) If v is a choice variable, we have the following problem,

$$\pi(p,w) = \max_{p,v} p \cdot D(p) - cv^2 D(p)$$

$$= \max_{p,v} (p - cv^2)(1 - \frac{p}{v})$$

Our first order conditions are,

$$\frac{\partial \pi}{\partial p} = 1 - \frac{2p}{v} + cv = 0$$

$$\frac{\partial \pi}{\partial v} = cp + \frac{p^2}{v^2} - 2cv = 0$$

Replacing p^* from (a) in our FOC for v, we get,

$$3c^2v^2 - 4cv + 1 = 0$$

$$(cv - 1)(3cv - 1) = 0$$

$$\implies v = \frac{1}{c} \text{ or } v = \frac{1}{3c}$$

Since the profit for each unit has to be positive, that is, $p - cv^2 = \frac{v - cv^2}{2} > 0$,

$$\implies v^* = \frac{1}{3c}$$

(c) Social Planner's Problem

$$\max_{p,v} TS(p,v) = \max_{p,v} \int_{\frac{p}{v}}^{1} \left(\theta v - cv^2\right) d\theta$$

Where the consumer buys the good only if $\theta \geq \frac{p}{v}$, the utility is θv, the marginal cost of the producer is cv^2 and $\theta v - cv^2$ is the surplus of selling to type θ. Then,

$$TS(p,v) = \frac{1}{2}v\theta^2 - cv^2\theta|_{\frac{p}{v}}^{1}$$

$$= \frac{1}{2}v - \frac{1}{2}\frac{p^2}{v} - cv^2 + cvp$$

The first order conditions give,

$$\frac{\partial TS}{\partial p} = -\frac{p}{v} + cv = 0$$

$$\frac{\partial TS}{\partial v} = \frac{1}{2} + \frac{1}{2}\frac{p^2}{v^2} - 2cv + cp = 0$$

We solve for p^* and get the following equation in terms of v,

$$3c^2v^2 - 4cv + 1 = 0$$

$$\implies v^* = \frac{1}{3c} \text{ or } v^* = \frac{1}{c}$$

$$\implies p^* = \frac{1}{9c} \text{ or } p^* = \frac{1}{c}$$

$$\implies TS^* = \frac{2}{27}c \text{ or } TS^* = 0$$

Since $\frac{2}{27}c > 0$, we choose $v^* = \frac{1}{3c}$ and there is no distortion in quality.

Input market power

(a) The profit maximization problem can be written as,

$$\pi(p, w) = \max_{x_1, x_2} p(log(x_1) + log(x_2)) - w_1x_1 - w_2x_2$$

The first order conditions give,

$$\frac{p}{x_1} - w_1 = 0 \implies x_1^* = \frac{p}{w_1}$$

$$\frac{p}{x_2} - w_2 = 0 \implies x_2^* = \frac{p}{w_2}$$

(b) The problem becomes,

$$\pi(p, w) = \max_{x_1, x_2} p(log(x_1) + log(x_2)) - (w_1 + x_1^2)x_1 - (w_2 + 2x_2)x_2$$

The first order conditions give,

$$\frac{p}{x_1} - (w_1 + x_1^2) - x_1(2x_1) = 0$$

$$\frac{p}{x_2} - (w_2 + 2x_2) - 2 = 0$$

From here we conclude that,

$$\frac{p}{x_1} = w_1 + x_1^2 + 2x_1^2 > w_1$$

$$\frac{p}{x_2} = w_2 + 4x_2 > w_2$$

Therefore, both x_1^*, x_2^* decrease.