Department of Economics
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Final Exam
ECON6140: Spring 2024

Answer the following questions to the best of your ability. Points for each sub-question are given in parentheses.

To receive credit, you must show your work. An ideal answer will take a form similar to appendix material in a research paper. The logical argument should be made clearly and concisely. Moreover, each step should be introduced with enough words that the reader can understand its objective. Please take a few moments to read the problem setup carefully before beginning your responses.

Section	Score
Q1	/40
Q2	/10
Q3	/20
Q4	/15
Total	/85

1. **A search-theoretic model of advertising.**

Households

The economy is populated by a "large family" with preferences for consumption given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t) - \gamma S_t - \gamma N_t \right\}.$$

The household supplies two types of "effort": (i) labor effort, denoted N_t; and (ii) consumption search effort, a.k.a. "shopping", denoted S_t. Both types of effort impose the same constant marginal disutility, γ. Workers earn a wage W_t for every unit of N_t they supply to the market (so the labor market is not frictional.). On the other hand, finding a unit of consumption requires forming a match with a good unit that is for sale.

A unit of search effort delivers a goods match with probability P_t^h, which the household takes as given, so that consumption from the household perspective evolves according to

$$(1-\delta)C_{t-1} + P_t^h S_t = C_t. \tag{1}$$

The household purchases consumption at price $P_t \equiv 1$, so that the household budget constraint is

$$W_t N_t + \pi_t = C_t, \tag{2}$$

where π_t corresponds to the profits earned by all the firms in the economy. You can assume that households do not trade firm shares.

Firms

A representative firm hires workers to produce output, according to the production function

$$Y_t = A_t N_t^{\alpha}, \tag{3}$$

where $\alpha \in (0,1)$. the firm hires workers in a competitive market. Before they can sell output, however, they must use up some of their output in order to advertise their products. Each unit of advertising, D_t, yields a match with probability P_t^f, so that the amount of goods the firm can sell evolves according to

$$C_t = (1-\delta)C_{t-1} + P_t^f D_t. \tag{4}$$

Firm profits π_t consist of the value of goods produced minus the cost of labor and an output cost of μ paid for each unit of advertising. The firm's objective is to maximize the discounted present value of profits, given by

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\lambda_{2t}}{\lambda_{20}} \pi_t \right\} \tag{5}$$

where λ_{2t} is the marginal utility of an additional unit of income in period t, which firm takes as exogenous.

Market clearing

Equilibrium goods matches are determined by an aggregate matching function $M(D_t, S_t) = \chi D_t^{\varepsilon} S_t^{1-\varepsilon}$. Each good adverstised has an equal chance of being matched, so that in equilibrium

$$P_t^f = M(D_t, S_t)/D_t = \chi(D_t/S_t)^{\varepsilon-1}. \tag{6}$$

Conversely,

$$P_t^h = M(D_t, S_t)/S_t = \chi(D_t/S_t)^{\varepsilon}. \tag{7}$$

In equilibrium, any good produced is either consumed or used up to pay for advertising, so that

$$C_t = Y_t - \mu D_t. \tag{8}$$

Finally technology is purely exogenous, and evolves according tos an AR(1) process in logs,

$$log(A_t) = \rho log(A_{t-1}) + \epsilon_t. \tag{9}$$

(a) In the language of the course, list separately the exogenous state variables, the endogenous state variables, and the endogenous control variables in this model. Finally, make a list of all of the exogenous parameters of this economy. (5 points)

(b) Write the household's Lagrangian optimization problem and find the first order necessary conditions for optimality of the household. Denoting the multipliers on constraints (1) and (2) with $\lambda_{1,t}$ and $\lambda_{2,t}$, respectively. Using your conditions, prove that (10 points)

$$\frac{\gamma}{P_t^h} = C_t^{-1} - \frac{\gamma}{W_t} + (1-\delta)\beta E_t\left[\frac{\gamma}{P_{t+1}^h}\right] \tag{10}$$

(c) Interpret equation (10) above in words using a marginal cost = marginal benefit intuition. (5 points)

$$\frac{\gamma}{P_t^h} = C_t^{-1} - \frac{\gamma}{W_t} + (1-\delta)\beta E_t\left[\frac{\gamma}{P_{t+1}^h}\right] \tag{11}$$

(d) Write the social planner's Lagrangian optimization problem and find the first order necessary conditions for optimality. Denote the multipliers on the relevant constraints $\theta_{i,t}$ for $i = 1, 2, ...$ (10 points)

(e) Now write the Bellman equation that corresponds to the social planner's optimization problem in this economy and find the first order necessary conditions for optimality using the envelope theorem. Show that the conditions from (1.c) and (1.d) are equivalent. (10 points)

2. **Solution I.** Here we are going to take a few steps towards solving the decentralized version of the model.

   (a) Log-linearize equation (10) from first principals. You should log-linearize the equation around the steady-state, but you may treat the steady-state values of endogenous variables as parameters. (5 points)

   (b) Using the log-linearized equation, compute the corresponding rows of the F_x, F_y, F_{xp}, F_{yp} matrices that would be required for the log-linearization solution procedure we used in class. (5 points)

3. **Solution II.** Here we are going to take some steps towards solving the planner's version of the economy.

   (a) Using pseudo-code, describe an algorithm that solves for the approximate numerical value function you found in (1.d) over a finite grid of points `cgrid` and `agrid`. Below, I proved some initial steps. Your code does not need to compile, but you should pay special attention to indexing, so that a naive programmer could implement your algorithm. Also, be sure to test for convergence of your iterations. Do not include a "policy iteration" step in your algorithm. (20 points)

File: solve_vf.m

```
params = parameters; %Copies parameters into workspace
model_ss(params);    %Copies steady-state values for A, K, C into workspace

%Capital grid
cgrid = linspace(.8*css, 1.2*css,nk);
agrid = linspace(.8*ass, 1.2*ass,na);


%Initial guess for value function
vf = 0*kgrid


%Largest number of iterations
maxiter = 10000
```

(continued on next page)

```
%main loop to update value function
tt = 0;     %iteration counter
crit = inf; %initial convergence criterion

while tt < maxiter && crit > 1e-9
       %Now you complete in bluebook using pseudo code









end
```

4. **News shocks.** Consider the effects of anticipated productivity changes in the real-business cycle model we used for numerical examples in class. Using a combination of economic intuition and equations, explain why productivity news in this model cannot not provide a satisfactory theory of the business cycle comovements of macroeconomic variables.

   (a) You answer should

      i. include a brief description of the empirical facts that are relevant to your argument

      ii. include a discussion of the implications of the RBC model with news for these facts

      iii. clarify which are your arguments are generic (i.e. for all parameters) vs. quantitative (i.e. for reasonable parameters.)

      iv. suggest a model modification (or modifications) that would resolve the challenges you highlight, and explain why does so

   (15 points)