

**Problem Set #1**

Due September 13

1. Economists have observed that the default setting in retirement plans seems to affect employees' choices about retirement savings. Using the language from class here is how the observations occur. In each scenario presented to the employee, there are two alternatives: a default percentage of the employee's salary to be deducted and placed in a retirement plan and another percentage of the employee's salary to be deducted and placed in a retirement plan if the employee objects to the default option. So the employee can choose "not object" and get the default percentage deducted or "object" and get the other percentage deducted. In the following, each scenario is described by a pair consisting of object or not object and the percentage deducted in each case.

In scenario I, the alternatives are: "not object" and 5 percent; and, "object" and 0 percent. In scenario II, the alternatives are: "object" and 5 percent; and, "not object" and 0 percent.

- a. Many people choose the default (the alternative with not objecting) in both scenarios. This is often said to be inconsistent with rational choice. Setup a model of a rational decision maker and show that these choices are inconsistent with rational choice in your model.
- b. In this part of the question, we want to ask if the conclusion in part (a), that the choices are inconsistent with rational choice, necessarily follows from the description of the decision problem as it is presented to the employee. To ask this question you need to describe a set of items,  $X$ , that is being considered by the employee and ask if there is a rational preference relation on  $X$  that could yield these choices. There are two possible answers. First, there does not exist an  $X$  and a rational preference relation on  $X$  consistent with these choices. Second, there does exist such an  $X$  and a rational preference relation on  $X$  consistent with these choices. If you believe that the first is true then prove it; if you believe that the second is true provide an example and show that it works.
- c. In this part of the question we want to ask what happens if additional scenarios are introduced. In scenario III, the alternatives are: do not object and 10 percent; and, object and 5 percent. In scenario IV, the alternatives are: do not object and 10 percent; and object and 20 percent. In scenario V, the alternatives are: do not object and 0 percent; and object and 20 percent. The employee's choices are observed to be: In III---object and 5 percent; In IV---do not object and 10 percent; and, In V---object and 20 percent. Is there a set of alternatives,  $X$ , and a rational preference relation on  $X$  that could yield all of the choices we observe in the five scenarios? Again, there are two possible answers. First, there does not exist an  $X$  and a rational preference relation on  $X$  consistent with these choices. Second, there does exist such an  $X$  and a rational preference relation on  $X$  consistent with

these choices. If you believe that the first is true then prove it; if you believe that the second is true provide an example and show that it works.

2. A consumer has a preference relation  $\succeq$  on  $R_+^1$  of the form  $x \succeq y$  if and only  $x \geq 2y$ . Is  $\succeq$  a rational preference relation? Explain briefly.
3. Let  $X$  be finite set of alternatives. Suppose  $\succeq$  is a rational preference relation on  $X$  and let  $C^*(\cdot, \succeq)$  be the choice function on  $X$ . Suppose that there are alternatives  $x, y \in X$  such that  $y \succ x$ .
  - a. Compare  $C^*(B, \succeq)$  and  $C^*(B - \{x\}, \succeq)$  for a set of alternatives  $B$  containing both  $x$  and  $y$ .
  - b. Compare  $C^*(B, \succeq)$  and  $C^*(B - \{x\}, \succeq)$  for a set of alternatives  $B$  containing  $x$ , but not containing  $y$ .
4. Let  $X = \{a, b, c\}$  be a set of alternatives and suppose that  $(B, C(\cdot))$  is a choice structure for which  $B$  is all non-empty subsets of  $X$ . Suppose that the choice structure satisfies WARP. You know that  $C(\{a, b, c\}) = \{a\}$  but you have no other direct information about  $C(\cdot)$ . What can you say about  $C(A)$  for the remaining  $A \in B$ ?
5. Let  $X$  be a finite, nonempty set and let  $\beta$  be all non-empty subsets of  $X$ , i.e.  $\beta = P(X)$ . Prove that any choice structure  $(\beta, C(\cdot))$ , with  $\beta = P(X)$ , that satisfies WARP satisfies Sen's  $\beta$ . [Sen's  $\beta$  for a choice structure  $(\beta, C(\cdot))$  is: For any  $A, B \in \beta$ , if  $x, y \in C(A)$ ,  $A \subset B$  and  $y \in C(B)$  then  $x \in C(B)$ .]
6. A consumer has preferences  $\succeq$  on  $R_+^n$  that can be represented by a quasi-concave utility function  $u : R_+^n \rightarrow R_+^1$ . You have been asked to describe the effect of a small tax on good one on the consumer's demand for good one. To do this you plan to start by solving the consumer's maximization problem. However, you don't like solving maximization problems with quasi-concave objective functions and so you plan to use a monotonic transformation  $f : R_+^1 \rightarrow R_+^1$  of the utility function to replace  $u : R_+^n \rightarrow R_+^1$  by  $v(x) = f(u(x))$  in the maximization problem. Is this valid? Will it give you the same demand as you would have found with the original utility function? Explain.
7. A consumer purchases goods  $x \in R_+^L$  with  $L \geq 2$  at prices  $p$  using wealth  $w$ . Let  $x^*$  be the consumer's chosen bundle of goods. You know that this consumer's choices satisfy Walras Law and WARP. Local authorities plan to use a tax on good 1 to discourage the consumption of good 1. Local authorities do not want the consumer to be harmed by this tax so they plan to give the consumer a subsidy that is just enough to make  $x^*$  affordable at the new prices  $p = (p_1 + t, p_2, \dots, p_L)$ , where  $t$  is the tax on good 1. The consumer treats this subsidy as a fixed number  $R$  that increases wealth to  $w+R$ . [Drawing a graph for the case of  $L=2$  is a good way to start on this problem, but it's not a complete answer as  $L$  is not necessarily 2.]

- (a) What happens to the amount of good 1 the consumer purchases? Explain briefly.
- (b) What would happen to the amount of good 1 the consumer purchases if there was no subsidy, i.e.  $R=0$ ? Explain briefly.
8. A consumer has a preference relation  $\succeq$  on  $[0,1]$  that is represented by the utility function  $U(x) = x - x^2$ . Is this consumer's preference relation convex? Explain briefly.
9. In year 0, a consumer has wealth  $w^0 = 1,000$ , prices are  $(p_1^0, p_2^0) = (10,10)$  and the consumer chooses  $(x_1^0, x_2^0) = (50,50)$ . In year 1, the consumer has wealth  $w^1 = 1,250$  and prices are  $(p_1^1, p_2^1) = (15,9)$ . For what range of choices of  $x_2$  can you conclude that the consumer's choices are inconsistent with the weak axiom? You can assume that the consumer's choices satisfy Walras Law.
10. One of your colleagues is interested in comparing the welfare of two people who live in locations where there are different prices for goods and where the two individuals have different wealths. Your colleague believes that these two people have common preferences. Specifically, he assumes that there are two consumers, 1 and 2, with rational and locally non-satiated preferences  $\succeq$  over consumption goods in  $\mathfrak{R}_+^L$ . The prices and wealths for consumers 1 and 2 are  $(p^1, w^1)$  and  $(p^2, w^2)$ , respectively. Each consumer selects a bundle of goods in their budget set that is best according to their preferences. Let these bundles be  $x^1$  and  $x^2$ .
- (i) Your colleague asks you to suggest how to interpret data that he might find about prices, wealths and choices. Specifically, he asks for each case below whether you can say that consumer 1 is better off than consumer 2 or consumer 2 is better off than consumer 1.
- $p^1 x^2 < w^1$  and  $p^2 x^1 > w^2$ .
  - $p^1 x^2 > w^1$  and  $p^2 x^1 > w^2$ .
  - $p^1 x^2 < w^1$  and  $p^2 x^1 < w^2$ .

For each of these cases what can you say who is better off? Explain briefly.

- (ii) Another colleague argues that this entire project (inferring who is better off from these choices) is flawed. This colleague makes the following argument:
- These two people choose where to live.
  - Suppose that they each were free to choose either location (there is no cost associated with this choice) and that all attributes of the locations that these people care about are reflected in the consumption goods.
  - Thus, each person prefers (at least weakly) the location they are in to the other location.
  - Then as they made different location choices either they have different preferences or at least one of them is indifferent between the locations.

- e. Thus, the assumption of common preferences is flawed, and if they don't have common preferences nothing, other than the fact that each consumer prefers their own consumption bundle to the one chosen by the other consumer, can be inferred from the choices over consumption bundles given locations.

Briefly evaluate this argument.