ECON 6090 - TA Section 8

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Exercises

1. [2016 Prelim 2] The set of prizes is $X = \{-1, 0, +1\}$ and a probability on these prizes is denoted by $p = (p_1, p_2, p_3)$. An individual strictly prefers a "small gamble", $p = (1/8, 3/4, 1/8)$, to certainty, $p = (0, 1, 0)$. However, the individual strictly prefers certainty to the "large gamble", $p = (1/2, 0, 1/2)$. Do this person's preferences have an objective expected utility representation? Explain.

2. [2014 June Q] An individual has to decide how much of her wealth $w > 0$ to invest in a risky asset. This asset will have positive rate of return r with probability p, or a negative rate of return l with probability $1 - p$. So if the individual invests x dollars in the risk asset, the with probability p her wealth will be $w - x + (1 + r)x$ and with probability $1 - p$ her wealth will be $w - x + (1 + l)x$. Assume that the asset has a strictly positive expected rate of return $pr + (1 - p)l > 0$. Feasible investments in the risky asset are $x \geq 0$. Assume that this individual is an expected utility maximizer with Bernoulli payoff function $u(w)$ with $u' > 0$ and $u'' < 0$ for all non-negative wealths.

   a. Show that the individual will invest a positive amount of wealth $x > 0$ in the risky asset.

   b. It seems reasonable to suppose that as an individual's wealth increases he would invest more in the risk asset. Whether this is true or not depends on how the individual's risk aversion changes as their wealth changes. What is this relationship? This is, under what conditions on risk aversion does investment in the risky asset increase as wealth increases? [Hint: Absolute Risk Aversion of utility function $u(\cdot)$ at X is equal to $-u''(x)/u'(x)$]

3. [2022 Prelim 2] An individual has initial wealth w^* and Bernoulli utility function $\ln(w)$, where w is wealth. The individual can invest initial wealth in any fractions in two projects: A and B. Any initial wealth not invested in A or B disappears. Let β be the fraction of initial wealth invested in project A and $1 - \beta$ be the fraction of initial wealth invested in project B. Exactly one of these projects will succeed; the other project fails and has payoff 0. With probability α project B fails and project A succeeds and pays off $p_A > 0$ dollars for every dollar invested in it. With probability $1 - \alpha$, project A fails and project B succeeds and pays off $p_B > 0$ dollars for

every dollar invested in it. [For example, if project A succeeds then an individual who invested βw^* in project A will have wealth $w = \beta w^* p_A$.]

a. Find the optimal value of β.

b. How does increasing p_A affect the amount invested in project A? Why?

c. Another individual is offered the same projects as above. This individual has Bernoulli utility for wealth $\ln(w^{1/2})$. Is this individual more risk averse than the one above? How does this individual's optimal investment rule compare to the one in (1)?