Midterm 1

ECON 6170

Instructions: You have the full class time to complete the following problems. You are to work alone. This test is not open book. In your answers, you are free to cite results that you can recall from class or previous homeworks *unless explicitly stated otherwise*. The exam is out of 20 points.

1. (4pts) Let $f, g : \mathbb{R} \to \mathbb{R}$ and $a \in \mathbb{R}$ be such that $f(a) = g(a)$.

   Let $h(x) = f(x)$, for all $x \leq a$ and $h(x) = g(x)$ for $x > a$.

   If f and g are continuous at a, prove that h is continuous at a.

2. (4pts) Let $X \subset \mathbb{R}^k$ and suppose that $f : X \to \mathbb{R}$ is continuous. Prove or disprove the following

   (a) If X is open, $f(X)$ is open.

   (b) If X is closed, $f(X)$ is closed.

   (c) If X is bounded, $f(X)$ is bounded.

   (d) If X is compact, $f(X)$ is compact.

   Note: Recall that $f(X) = \{f(x) | x \in X\}$.

3. (4pts) Suppose $f, g : [0,1] \to \mathbb{R}$ are continuous, and $f(0) < g(0)$, and $f(1) > g(1)$. Then prove that $f(x) = g(x)$ for some $x \in [0,1]$.

4. (4pts) Suppose $f : A \subset \mathbb{R}^k \to \mathbb{R}^n$ is continuous and suppose A is convex. Prove or disprove that $f(A)$ is convex when:

   (a) $k = n = 1$.

   (b) $k = n = 2$.

   Hint: Draw pictures if it helps.

5. (4 pts) Recall that the closure of a set $X \subset \mathbb{R}^k$, denoted $\bar{X}$, is the intersection of all the closed sets containing X.

   Call $x' \in \mathbb{R}^k$ a limit point of X if there is some sequence $(x_n)_n$, with $x_n \in X - \{x'\}$ for all $n \in \mathbb{N}$, such that $x_n \to x'$.

   Let $\hat{X}$ be the union of X and the set containing all the limit points of X.

   Prove that $\hat{X} = \bar{X}$.

   Note: x' may or may not be an element of X, but at any rate, $X - \{x'\}$ is the set of all points in X except x'.