ECON 6190

Problem Set 0

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1. The probability that someone will receive a speeding ticket is equal to one minus the probability that they will not be caught at all of the locations. In other words, the probability is

$$1 - (0.92 \cdot 0.97 \cdot 0.9 \cdot 0.94) = 0.245$$

2. We have that $P\{I\} = 0.0025$, that $P\{T \mid I\} = 0.9$, and that $P\{T \mid I^c\} = 0.01$. By Bayes' Rule:

$$P\{I \mid T\} = \frac{P\{T \mid I\}P\{I\}}{P\{T \mid I\}P\{I\} + P\{T \mid I^c\}P\{I^c\}} = \frac{0.00225}{0.00225 + 0.009975} = 0.184$$

3.

4.

5. We have that

$$f_{XY}(x,y) = \begin{cases} xe^{-y} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) $f_X(x) = \int_x^\infty xe^{-y}dy = -xe^{-y} \mid_x^\infty = 0 - (-xe^{-x}) = xe^x$, for $x > 0$.

Then we have that

$$X_{Y|X}(y \mid x) = \frac{f_{XY}(x,y)}{f_X(x)} = \begin{cases} e^{x-y} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(b) $\mathbb{E}[Y \mid X = x] = \int_x^\infty y f_{Y|X}(y \mid x)dy = \int_x^\infty ye^{x-y}dy = e^x \int_x^\infty ye^{-y}dy$

Need integration by parts. We get

$$\mathbb{E}[Y \mid X = x] = e^x \left\{ y(-e^{-y}) \mid_x^\infty - \left(- \int_x^\infty e^{-y}dy \right) \right\} = e^x\{xe^{-x} + e^{-x}\} = x + 1$$

(c) We have that

$$var(Y \mid X = x) = \mathbb{E}[(Y - \mathbb{E}[Y \mid X = x])^2 \mid X = x] = \mathbb{E}[Y^2 \mid X = x] - (\mathbb{E}[Y \mid X = x])^2$$

(full solution in notes, but)

$$var(Y \mid X = x) = X^2 + 2X + 2 - (X + 1)^2 = 1$$

(d) They are not – since $\mathbb{E}[Y \mid X] = x + 1 \neq \mathbb{E}[Y]$, from part (b), they are not independent.