Economics 6090 Professor Philipp Kircher

Fall 2023 Cornell University

Problem Set #2

DUE Friday, October 4, 1.25pm
(Please submit answers online in Canvas)

1. Study Berge's Maximum Theorem, and make a short video (no longer than 2 minutes) in which you explain its basic insight and intuition in your own words using a single graph that you share on screen. (An easy way to record a presentation is to present it in Cornell zoom and record your presentation in the cloud).

   Upload a link to your recording on Canvas. The referees will grade a randomly selected 10 of the submissions – for the others only the remaining exercises count.

2. Imagine a worker who chooses how much to work (t) and how much to consume (x), where each of them is a non-negative scalar. The worker earns a wage normalized to 1 per hour worked, but only has w hours available. The consumers utility function of work and consumption is given by $u(x,t) = (x^{\frac{1}{2}} + (w-t))^2$. Assume that the hours budget w and the price of the consumption good p are the only free variables in the model (i.e., no changes in the wage rate).
   a. Write the workers problem in a more conventional way by writing his utility function in terms of consumption x and leisure l, given a budget constraint.
   b. Find the worker's Walrasian demand functions for goods x and l.
   c. Find the worker's indirect utility function using the utility function given in the statement of this problem.
   d. Suppose that time endowment w and price p are such that the worker chooses some strictly positive level of leisure. Find the worker's Hicksian demand function for good x for price and utility levels consistent with strictly positive purchases of leisure.

3. Consider a consumer with an expenditure function that e(p,u) that is multiplicatively separable in the sense that e(p,u)=g(u)r(p) for some strictly increasing function g(u) and strictly increasing function r(p).
   a. Find this consumer's Walrasian demand function.
   b. Exploit Walras Law to show that $r(p) = \sum_{i=1}^{L} p_i \frac{\partial r(p)}{\partial p_i}$. Do you need to make any assumptions on g(u) to arrive at this equality?
   c. Now suppose there is a finite number M of consumers in the economy that all share this expenditure function but that might not have the same budget. Does the distribution of budgets matter for aggregate demand? (If not, it means that you created a representative agent economy).

4. Consider a consumer that makes choices how much to buy of two different products given a budget constraint. You happen to know that the expenditure function of the consumer it is of form $e(p, U) = U p_1^{\alpha} p_2^{\beta}$.
    a. What restrictions (if any) on the parameters α and β are required to ensure that e(.) constitutes a valid expenditure function. Assume that the restriction(s) are/is satisfied.
    b. Find the indirect utility function, the Hicksian demand functions, and the uncompensated demands. Carefully list the correct arguments for each function.
    c. Use an alternative approach — i.e. different to what you did in b) — to calculate the uncompensated demand functions.
    d. Assume the consumer has α=β=1/2, and a budget of 512. Assume environmental legislation increases prices from $p_1 = p_2 = 1$ to $p_1 = p_2 = 16$. One way to think about how to assess the loss of welfare for this consumer due to the price increase is to ask how much money would one have to give this consumer to be equally well off. There are two ways of doing this.
        i. What is the compensating variation for this consumer? What is the equivalent variation for this consumer?
        ii. If compensating variation and equivalent variation differ, explain why one is higher than the other in an intuitive way. Which one would you think is more reasonable if one intended to pay this consumer for his consent to agree to the price increase?

5. Evaluate the following (explain your answer):
    a. Consider utility function $u(x) = 2\ln(x_1) + 2\ln(x_2)$ and associated expenditure function $e(u, p_1, p_2)$. Now consider utility function $u^*(x) = x_1 x_2$ with associated expenditure function e^*. Claim to evaluate: $e(u, p_1, p_2) = e^*(u^*, p_1, p_2)$ if $u^* = \exp\left(\frac{u}{2}\right)$.
    b. Consider a consumer with continuous and locally non-satiated preferences and income w who consumes strictly positive amounts of all goods at a given price vector p>>0. Now the price of good i increases from p_i to p_i', while all other prices stay the same. Assume his income increases by $(p_i' - p_i)x_i(p, w)$. Claim to evaluate: this consumer always purchases less of good i under the new prices compared to the old prices, and obtains a higher utility under the new prices.
    c. A consumer who will live for $T \geq 2$ periods has utility function $\sum \beta^t u(c_t)$ for consumption path $(c_1, \ldots, c_T)$. Assume that $0 < \beta < 1$, $u'(c) > 0$ and $u''(c) < 0$ for all $c \geq 0$, and $\lim_{c \to 0} u'(c) = +\infty$. Consumption in each period must be non-negative and total consumption can be no more than wealth, $w > 0$, i.e. $\sum c_t \leq w$. Claim to evaluate: Optimal consumption c_t^* can increase under some utility functions.

6. Find an (interesting?) research paper in industrial organization, labor economics, health economics, or some other area of economics that relies on the insights from consumer theory that we discussed in class. Which insights in particular are they using?