ECON 6090 - TA Section 6

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1. Solving problems with continuum of inputs

Suppose a single-output firm takes as input a continuum of commodities $j \in [0,1]$. The production function is

$$f(z) = \int_0^1 z(j)^\alpha dj$$

where $\alpha \in (0,1)$. Find the unconditional input demand function $x(j,p,w)$ where $w : [0,1] \to \mathbb{R}^+$ is a continuous function integrable on $[0,1]$.

2. A question from a past Q exam

A firm produces output y using the production function $y = x_1^\alpha x_2^\beta$ where $x_1, x_2 \geq 0$ are inputs and $\alpha, \beta > 0$, $\alpha + \beta < 1$. Input prices are $w_i > 0$ for input i. The output price will be either $p_1 > 0$ or $p_2 > 0$. The probability of output price p_1 is δ where $0 < \delta < 1$, and of course the probability of output price p_2 is $1 - \delta$. This firm chooses output to maximize expected profit and it knows the production function, input prices, and distribution of output prices.

(a) Suppose that the firm has to choose how much to produce before knowing the realization of the output price. What is the optimal output?

(b) Suppose that the firm first observes the realization of the output price and then decides how much to produce. How much will it produce if the price is p_1; how much will it produce if the price is p_2?

(c) Is the following conjecture true or false: If $\alpha + \beta = \frac{1}{2}$, then the expectation of the outputs in part (b) equals the output in part (a). Explain briefly.

(d) Generally, is the expectation of the profits in part (b) greater than, equal to, or less than the profit in part (a)? Explain briefly. [Do not assume that $\alpha + \beta = \frac{1}{2}$.]

3. The cost function

Suppose a firm has constant marginal cost and $C(w, 0) = 0$. What do we know about the firm's production function? Give examples of at least three widely-used classes of production functions that could generate such a cost function.