ECON 6090-Microeconomic Theory. TA Section 2

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In Section notes

Consumer Choice Theory
Model:

(.) Goods: $x = (x_1, x_2, ..., x_L) \in \mathbb{R}^L_+$

(.) Price: $p = (p_1, p_2, ..., p_L) \in \mathbb{R}^L_{++}$

(.) Wealth: $w \in \mathbb{R}_{++}$

(.) Budget Set: $B_{p,w} = \{x \in \mathbb{R}_+, p \cdot x \leq w\}$

(.) Choice (Walrasian Demand): $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$

Assumptions:

1. $x(p, w)$ is HoD 0 in (p, w)
2. Walras Law: $p \cdot x(p, w) = w$

Remark: $(\mathcal{B}, x(.))$ will be the choice structure, where $\mathcal{B} := \{B_{p,w} : (p, w) \in \mathbb{R}^L_{++} \times \mathbb{R}_{++}\}$.

Weak Axiom of Revealed Preference (WARP):
$\forall (p, w), (p', w')$, if $p \cdot x(p', w') \leq w$ and $x(p, w) \neq x(p', w')$, then $p' \cdot x(p, w) > w'$.

This is equivalent, under assumptions 1 and 2, when $w' = p' \cdot x(p, w)$ (**Compensated price change**), to saying,

$$(p' - p)(x(p', w') - x(p, w)) \leq 0 \ (< 0 \text{ if } x(p, w) \neq x(p', w'))$$

Exercises

WARP and Consumer Choice

1. (2004 Prelim 1)

(a) By Walras Law,

$$p'_x x' + p'_y y' = w'$$

$$\implies x' = \frac{1250 - 9y'}{15}$$

And notice,

$$p'_x x^0 + p'_y y^0 = 1200 < 1250 = w'$$

To violate WARP, we must have,

$$p^0_x x' + p^0_y y' = 10y' + \frac{2500 - 18y'}{3} \leq w^0 = 1000$$

$$y' \leq \frac{125}{3}$$

Since $x' \geq 0, y' \geq 0$,

$$\implies y' \in [0, \frac{125}{3}]$$

2. (2016 Prelim 1)

(a) By Walras Law,

$$p^a \cdot x^a = w^a$$
$$\implies x_1^a + x_2^a + 2x_a^3 = 13$$
$$\implies x_a^3 = 4$$

Similarly,

$$\implies x_b^3 = 8$$

Then,

$$p^a \cdot x^b > w^a$$
$$p^b \cdot x^a < w^b$$

Satisfies WARP.

(b) Now,

$$p^a \cdot x^a = w^a$$
$$\implies 10 + x_2^a + x_3^a = 20$$
$$\implies x_3^a = 10 - x_2^a$$

And,

$$p^b \cdot x^b = w^b$$
$$\implies 10 + x_2^b + 2x_3^b = 30$$
$$\implies x_3^b = 10 - \frac{1}{2}x_2^b$$

Since $x_a \neq x_b$, we must have at least one of the following two scenarios to hold to satisfy WARP.

1) $p^a \cdot x^b > w^a \implies x_2^b > 10$
2) $p^b \cdot x^a > w^b \implies x_2^a < 10$

Since $x_3^a \geq 0$ and $x_3^b \geq 0$, we also know (3),

$$0 \leq x_2^a \leq 10$$
$$0 \leq x_b^2 \leq 20$$

Finally, putting all the information together,

$$(x_2^a, x_2^b) \in [0, 10) \times [0, 20] \cup [0, 10] \times (10, 20]$$

Where $[0, 10) \times [0, 20]$ comes from adding cases (2) and (3), and $[0, 10] \times (10, 20]$ comes from adding cases (1) and (3).[1]

[1] Another way to think about it is: $(1 \cup 2) \cap 3 = (1 \cap 3) \cup (2 \cap 3)$