

# ECON 6090 - TA Section 10

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## Exercises

1. (Qualifying 2022). An individual is offered an opportunity to bet on the toss of a coin. The individual begins with wealth  $\bar{w} > 0$ . If she guesses correctly (the coin lands heads and she guessed heads or the coin lands tails and she guessed tails), her wealth increases to  $\beta\bar{w}$  and if she guesses wrong her wealth declines to  $\alpha\bar{w}$ , where  $0 < \alpha < 1 < \beta$ . Her payoff function for any amount of wealth,  $w$ , is  $u(w) = \ln(w)$ . The individual does not know the probability of heads (or tails). It could be either  $1/3$  or  $2/3$ . Lets call the state in which probability of heads is  $2/3$  state  $H$  and the state in which the probability of heads is  $1/3$  state  $T$ . [Note that  $H$  and  $T$  do not refer to outcomes of a coin toss. They are states of the world in which heads is more likely (state  $H$ ) or heads is less likely (state  $T$ ).]
  - a. Suppose that this person is a subjective expected utility maximizer and that she believes that the probability of state  $H$  is  $1/2$ . The probability of state  $T$  is also  $1/2$ . For what values of  $\alpha$  and  $\beta$  would she be just indifferent between accepting and rejecting the bet?
  - b. Suppose now that this person does not know the probability of states  $H$  or  $T$ . She does not have a probability on these states. Instead, this individual is an ambiguity averse decision maker (of the Gilboa-Schmeidler type). For what values of  $\alpha$  and  $\beta$  would she be just indifferent between accepting and rejecting the bet?
  - c. One of your colleagues states that: In part (a) your answer would have been unaffected by changing the probability of heads in state  $H$  to  $9/10$  and probability of heads in state  $T$  to  $1/10$ . (And of course keeping the subjective probability of state  $H$  at  $1/2$ .) But for the ambiguity averse agent in part (b) this change would change the answer to question (b). That is, changing the probability of heads in state  $H$  to  $9/10$  and probability of heads in state  $T$  to  $1/10$  would change the answers for an ambiguity averse agent.  
Is your colleague's claim correct? Explain. It is not enough to just repeat the equations as an explanation. You need to provide an explanation in words of why your colleague's observation is correct or incorrect.

2. (Final 2021)

- a. An individual is offered an opportunity to bet on the flip of a coin. The outcome of the flip will be Heads or Tails. If the individual bets on H and H occurs or T and T occurs the individual wins \$1, if the bet is on H and T occurs or if it is on T and H occurs the individual loses \$1. So a correct bet wins \$1 and an incorrect bet loses \$1. If the individual does not bet there is no change in wealth. The probability of H occurring is  $1/3$  in state A and  $3/4$  in state B. The individual does not know the true state. This individual is a subjective expected utility maximizer with probability of  $1/2$  on each of the states, A and B. The individual is risk neutral, i.e., the utility function defined on wealth,  $w$ , is  $u(w) = w$ . Will the individual bet and if so will the bet be on H or on T ? Explain or prove your answer.
- b. In the setup of the problem above suppose that the individual is Gilboa-Schmeidler ambiguity averse person. Everything is the same as the setup above except that the individual does not have a prior on states and is ambiguity averse rather than being a SEU maximizer. The range of probabilities on state A that the individual consider to be possible is  $[1/3, 2/3]$ . Will this individual bet? Explain or prove your answer.